

Criterion for Local Thermal Equilibrium in Forced Convection Flow Through Porous Media

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ABSTRACT

In this paper, a general criterion for local thermal equilibrium is presented in terms of parameters including the effective fluid Prandtl number, the particle Reynolds number, the effective solid-to-fluid thermal conductivity ratio, the Darcy number, the Nusselt number, and porosity. In order to check the validity of the proposed criterion for local thermal equilibrium, the forced convection phenomena in the porous medium between two parallel plates subjected to constant temperature are studied by a numerical method based on the Brinkman–Forchheimer extended Darcy model. The proportion of temperature difference between solid and fluid phases in a representative elementary volume to the temperature rise of fluid is studied by comparing the effects of relevant parameters in this new criterion. In addition, the proposed criterion is consistent with the existing experimental and numerical results for convection heat transfer in porous medium.

NOMENCLATURE

<p>a_{sf} specific surface area</p> <p>C_f fluid specific heat</p> <p>D distance of parallel</p> <p>Da Darcy number</p> <p>d_p diameter of particle</p> <p>F geometric function</p> <p>h_{sf} interfacial heat-transfer coefficient</p> <p>K permeability</p> <p>L characteristic length for system</p> <p>Nu Nusselt number</p> <p>p pressure</p> <p>Pr Prandtl number</p> <p>q_w boundary heat flux (W/m²)</p> <p>q_{sf} the volumetric rate of interfacial heat transfer</p> <p>Re_{dp} particle Reynolds number</p> <p>S flow section area</p>	<p>T temperature</p> <p>t time</p> <p>u, v effective velocity components</p> <p>V volume of porous medium</p> <p>Greek</p> <p>η viscosity of fluid φ</p> <p>κ effective solid-to-fluid thermal conductivity ratio ρ</p> <p>λ conductivity of fluid or solid η</p> <p>ρ density λ</p> <p>φ porosity κ</p> <p>Subscripts</p> <p>eff effective</p> <p>f fluid</p> <p>in inlet</p> <p>s solid</p> <p>w wall</p>
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1. INTRODUCTION

Forced convection heat transfer in porous media has been extensively investigated in past decades. This interest stems from the variety of engineering applications associated with the presence of porous medium in the path of the working fluid. In essence, the presence of a complicated solid matrix with a high thermal conductivity significantly increases the capability of the system to transport energy. A crucial parameter in dealing with heat transfer in porous media lies in treating the local temperature difference between fluid and solid. The literature frequently cites two approaches in investigating the energy transport in porous media based on the volume-averaging method. The first one is the local thermal equilibrium (LTE) model, and the other is the local thermal nonequilibrium (LTNE) model. The LTE model assumes the temperature of solid phase is equal to that of fluid

phase in a representative elementary volume (REV), which is simple and straightforward, but valid only when the temperature difference between the solid and fluid phases is very small. The LTNE model requires additional information to describe the way of energy transportation between the two phases.

A number of investigations have been reported on the validity of the LTE assumption in the forced convection channel flow. Carbonell and Whitaker (1984) presented a criterion for the validity of the LTE assumption. Their criterion was proposed in the case where the effect of conduction is dominant. Amiri and Vafai (1994) presented an error contour map in terms of the particle Reynolds number, the Darcy number, and the ratio of thermal diffusivities based on the qualitative ratings for forced convection through a packed bed channel. Nield (1998) concluded that the effect of local thermal nonequilibrium is to reduce the Nusselt number at the interface between the fluid and

solid phases. Lee and Vafai (1999) proposed a criterion for the validity of the LTE model in the case of flow through a porous channel subjected to a constant heat flux on the top and bottom walls by using analytical solutions based on the Darcian flow model. Kim et al. (2000) showed that the LTE assumption in a microchannel heat sink, which is modeled as a porous medium, is valid as the Darcy number approaches zero and the effective fluid-to-solid thermal conductivity ratio infinity. Kim and Jang (2002) also presented a criterion in terms of Prandtl number, Reynolds number, and Darcy number, but not including the effect of effective solid-to-fluid conductivity ratio. Although studies on the LTE model have been conducted for many years, to the authors' knowledge, the criterion for the validity of the LTE assumption has to be improved.

The aim of this study is to present a more general criterion for the LTE assumption. To do this, a theoretical analysis is performed for the case when the effect of convection heat transfer is dominant in a channel filled with particles. In order to check the validity of the new criterion for the LTE assumption, the steady-state incompressible flow through a porous bed between two parallel plates subjected to constant temperature is studied by using a numerical method based on the Brinkman–Forchheimer extended Darcy model. The effects of Prandtl number, Reynolds number, effective solid-to-fluid conductivity ratio, Darcy number, and Nusselt number on the proportion of temperature difference between solid and fluid phases in a REV to the temperature rise of fluid are studied systematically.

2. CRITERION FOR LOCAL THERMAL EQUILIBRIUM

Kaviany (1995) pointed out that the LTE assumption is valid when the temperature difference between the solid phase and the fluid phase in a REV is much smaller than the fluid temperature difference over the system dimension

$$\Delta T_L \gg \Delta T_l \quad (1)$$

The temperature difference between the solid and fluid phases in the REV can be expressed as

$$\Delta T_l = T_s - T_f = \frac{q_{sf}}{h_{sf} \alpha_{sf}} \quad (2)$$

where T_s and T_f are the intrinsic volume-averaged temperature of the solid phase and that of the fluid phase, respectively. q_{sf} , h_{sf} , and α_{sf} are the volumetric rate of interfacial heat transfer, the interfacial heat transfer coefficient, and the specific surface area of the packed bed, respectively.

The fluid temperature difference over the system dimension can be expressed as

$$\Delta T_L = T_{f,\text{outlet}} - T_{f,\text{inlet}} = \frac{Q}{\rho_f C_f \dot{m}} \quad (3)$$

where $T_{f,\text{outlet}}$, $T_{f,\text{inlet}}$, Q , and \dot{m} are the outlet temperature of fluid, the inlet temperature of fluid, the heat-transfer rate carried by the fluid flowing through the porous medium, and the mass flow rate, respectively. The amount of heat transferred to or from the fluid through the porous medium can be expressed as

$$Q = \int_0^L q_w \quad (4)$$

On the boundary, the part of heat is directly transferred to the fluid. The rest is transferred to the solid phase first, and then to the fluid,

$$q_w = q_{ws} + q_{wf} \quad (5)$$

The boundary heat flux to the solid phase can be expressed as

$$q_{ws} = -\lambda_{s,\text{eff}} \left(\frac{\partial T_s}{\partial y} \right)_w \quad (6)$$

The boundary heat flux to the fluid phase can be expressed as

$$q_{wf} = -\lambda_{f,\text{eff}} \left(\frac{\partial T_f}{\partial y} \right)_w \quad (7)$$

If assume that (Alazmi and Vafai, 2002)

$$\left(\frac{\partial T_s}{\partial y} \right)_w = \left(\frac{\partial T_f}{\partial y} \right)_w \quad (8)$$

then

$$q_{ws} = q_w \frac{\lambda_{s,\text{eff}}}{\lambda_{s,\text{eff}} + \lambda_{f,\text{eff}}} = q_w \frac{\kappa}{\kappa + 1} \quad (9)$$

where κ is the solid-to-fluid effective thermal conductivity ratio defined as

$$\kappa = \frac{\lambda_{s,\text{eff}}}{\lambda_{f,\text{eff}}} \quad (10)$$

so the volume average temperature differences can be written as

$$\Delta T_l = \frac{\int_0^L q_{ws}}{h_{sf} \alpha_{sf} V} = \frac{\frac{\kappa}{\kappa+1} \int_0^L q_w}{h_{sf} \alpha_{sf} V} \quad (11)$$

and the fluid temperature difference as

$$\Delta T_L = \frac{\int_0^L q_w}{\rho_f C_f \dot{m}} \quad (12)$$

By substituting Eqs. (11) and (12) into Eq. (1), the criterion for LTE assumption can be expressed by the following equation:

$$R = \frac{\Delta T_l}{\Delta T_L} = \frac{\rho_f C_f \dot{m} \frac{\kappa}{\kappa+1}}{h_{sf} \alpha_{sf} V} \ll 1 \quad (13)$$

In Eq. (13), R represents the proportion of temperature difference between solid and fluid phases in a REV to the temperature rise of fluid. The interfacial heat-transfer coefficient h_{sf} and the interfacial surface area per unit volume α_{sf} can be expressed, respectively, as

$$h_{sf} = \frac{\text{Nu} \lambda_{f,\text{eff}}}{d_p} \quad (14)$$

and

$$\alpha_{sf} = \frac{6(1-\varphi)}{d_p} \quad (15)$$

where the specific surface area is calculated based on the particle parameter instead of the pore parameter. By substituting Eqs. (14) and (15) into Eq. (13), the LTNE number is expressed by the following equation:

$$R = \frac{\Delta T_l}{\Delta T_L} = \frac{\eta C_p \rho u_{in} d_p d_p \frac{\kappa}{L \kappa + 1}}{\lambda_{f,\text{eff}} \eta} \times \frac{1}{\text{Nu}} \frac{1}{6(1-\varphi)} \quad (16)$$

The effective fluid Prandtl number, the particle Reynolds number, the Darcy number, and the Nusselt number are defined, respectively, as

$$\text{Pr}_{f,\text{eff}} = \frac{\eta C_p}{\lambda_{f,\text{eff}}}, \quad \text{Re}_{d_p} = \frac{\rho u_{in} d_p}{\eta}$$

$$\text{Da} = \frac{d_p^2}{L^2}, \quad \text{Nu} = \frac{h_{sf} d_p}{\lambda_{f,\text{eff}}} \quad (17)$$

The new criterion for local thermal equilibrium is then gained as

$$R = \text{Pr}_{f,\text{eff}} \text{Re}_{d_p} \text{Da}^{1/2} \frac{\kappa}{\kappa + 1} \text{Nu}^{-1}$$

$$\times \frac{1}{6(1-\varphi)} \ll 1 \quad (18)$$

3. MATHEMATICAL VALIDATION OF THE CRITERION

3.1. Mathematical Formulation

The mathematical validation of the criterion is carried out for the two-dimensional steady-state flow through a parallel-plate channel with height D , length L , and aspect ratio $L/D = 10$. The channel is considered to be filled with homogenous and isotropic porous medium, as illustrated in Fig. 1. In addition, the porous medium is considered to have no spatial variation in porosity throughout the whole region, and

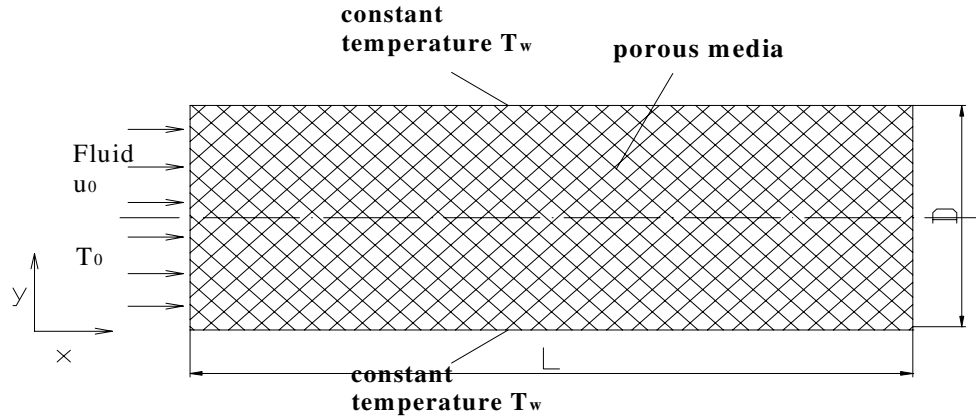


Figure 1. Schematic diagram of a porous bed

fluid is injected through the left inlet at a constant temperature with a uniform inlet velocity u_{in} .

The steady-state volume-averaged governing equations presented by Amiri et al. (1995) for the LTNE model are given as follows.

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (19)$$

Momentum of x direction:

$$\begin{aligned} \frac{\rho_f}{\phi} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial P}{\partial x} + \frac{\eta}{\phi} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\ -\frac{\eta}{k} u - \rho_f \frac{F\phi}{\sqrt{k}} \sqrt{u^2 + v^2} u & \end{aligned} \quad (20)$$

Momentum of y direction:

$$\begin{aligned} \frac{\rho_f}{\phi} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) &= -\frac{\partial P}{\partial y} + \frac{\eta}{\phi} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\ -\frac{\eta}{k} v - \rho_f \frac{F\phi}{\sqrt{k}} \sqrt{u^2 + v^2} v & \end{aligned} \quad (21)$$

Fluid phase energy:

$$\begin{aligned} \frac{\partial (C_f \rho_f u T_f)}{\partial x} + \frac{\partial (C_f \rho_f v T_f)}{\partial y} &= \frac{\partial}{\partial x} \left(\lambda_{f,eff} \frac{\partial T_f}{\partial x} \right) \\ + \frac{\partial}{\partial y} \left(\lambda_{f,eff} \frac{\partial T_f}{\partial y} \right) + h_{sf} \alpha_{sf} (T_s - T_f) & \end{aligned} \quad (22)$$

Solid phase energy:

$$\begin{aligned} \frac{\partial}{\partial x} \left(\lambda_{s,eff} \frac{\partial T_s}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_{s,eff} \frac{\partial T_s}{\partial y} \right) \\ - h_{sf} \alpha_{sf} (T_s - T_f) = 0 \end{aligned} \quad (23)$$

The permeability of the packed bed presented by Ergun (1952) and the geometric function F presented by Vafai (1984) are expressed as

$$K = \frac{\phi^3 d_p^2}{150 (1 - \phi)^2}, \quad F = \frac{1.75}{\sqrt{150} \phi^{3/2}} \quad (24)$$

The effective thermal conductivities of both phases are defined as

$$\lambda_{f,eff} = \phi \lambda_f, \quad \lambda_{s,eff} = (1 - \phi) \lambda_s \quad (25)$$

The no-slip boundary condition at the wall is employed for the momentum equation, while the boundary conditions for the energy equations are the constant wall temperature in the numerical analysis.

3.2. Results and Discussion

The numerical solution of the momentum and energy equations described above is obtained by using the finite volume method. The SIMPLE algorithm for the pressure-velocity coupling is used. Convergence is measured in terms of the maximum change in

each variable during iteration at each time increment. The maximum change allowed for the convergence check is set to 10^{-6} for the energy equation and 10^{-4} for both continuity and momentum equations. All computations have been carried out for a half parallel channel $L \times (D/2)$ by using 1000×100 nonuniform grid arrangements to ensure the results are independent of the grid system.

The influences of relevant parameters defined in Eq. (18) on R are indicated in Fig. 2 by numerical calculation. As showed in Fig. 2, R increases linearly with the increase of $\text{Pr}_{f,\text{eff}}$, Re_{d_p} , $\kappa/(\kappa+1)$, $\text{Da}^{1/2}$, Nu , and $1/(1-\varphi)$, which implies the assumption in Eq. (8) is reasonable. From the numerical calculation shown in Fig. 2, the effect of local thermal equilibrium in a porous medium is more obvious with the decrease of the effective fluid Prandtl number, the particle Reynolds number, the Darcy number, and the effective solid-to-fluid thermal conductivity ratio, as well as with the increase of the Nusselt number. In addition, when the effective solid-to-fluid thermal conductivity ratio is small, its effect is significant.

3.3. Discussion of Nusselt Number

The Nusselt number based on the interstitial heat-transfer coefficient is an independent parameter in the validation of the new criterion, determined by physical character of fluid and solid matrix, character of flow, as well as other parameters.

The Nusselt number, presented by Wakao et al. (1979), is expressed as

$$\text{Nu} = h_{\text{st}}d_p/\lambda_f = 2.0 + 1.1\text{Pr}^{1/3}\text{Re}_{d_p}^{0.6} \quad (26)$$

For the sintered metal, Kar and Dybbs (1982) suggested the Nusselt number as

$$\text{Nu} = \frac{h_{\text{st}}d_p}{\lambda_f} \sim C\text{Re}_{d_p}^n \quad (27)$$

$$(0 < \text{Re}_{d_p} < 10^2, \quad n \approx 1.35)$$

For the cellular ceramic, the Nusselt number is presented by Fu et al. (1998) as

$$\text{Nu} = \frac{h_{\text{st}}d_p}{\lambda_f} \sim C\text{Re}_{d_p}^n \quad (28)$$

$$(0 < \text{Re}_{d_p} < 10^3, \quad 0.9 < n < 1.18)$$

In those cases, the Nusselt number can also be expressed in the form of

$$\text{Nu} = B + C\text{Pr}^d\text{Re}_{d_p}^f, \quad 0 < d < 1, \quad f > 0 \quad (29)$$

So the criterion can also be written as

$$R = \text{Pr}_{f,\text{eff}}\text{Re}_{d_p}\text{Da}^{1/2} \frac{\kappa}{\kappa+1} \left(B + C\text{Pr}_{f,\text{eff}}^d\text{Re}_{d_p}^f \right)^{-1} \times \frac{1}{6(1-\varphi)} \ll 1 \quad (30)$$

When conduction becomes dominant in a porous medium as either $\text{Pr}_{f,\text{eff}} \rightarrow 0$ or $\text{Re}_{d_p} \rightarrow 0$, the Nusselt number approaches constant B . Then the criterion reduces to

$$R = \text{Pr}_{f,\text{eff}}\text{Re}_{d_p}\text{Da}^{1/2} \frac{\kappa}{\kappa+1} \frac{1}{B} \frac{1}{6(1-\varphi)} \ll 1 \quad (31)$$

When convection is dominant, Eq. (29) can be expressed as

$$\text{Nu} = C\text{Pr}^d\text{Re}_{d_p}^f, \quad 0 \leq d < 1, \quad f > 0 \quad (32)$$

Thus, the criterion can be written as

$$R = \text{Pr}_{f,\text{eff}}^{1-d}\text{Re}_{d_p}^{1-f}\text{Da}^{1/2} \frac{\kappa}{\kappa+1} \frac{1}{6(1-\varphi)} \ll 1 \quad (33)$$

$$0 \leq d < 1, \quad f > 0$$

Because $1-d > 0$, the assumption of local thermal equilibrium is valid as the Prandtl number is decreased, but the effect of Reynolds number is determined by f value. If $0 < f < 1$, $1-f > 0$, the assumption of local thermal equilibrium is valid as the Reynolds number decreases. If $f > 1$, $1-f < 0$, the assumption of local thermal equilibrium is valid as the Reynolds number increases. If $f = 1$, $1-f = 0$, the

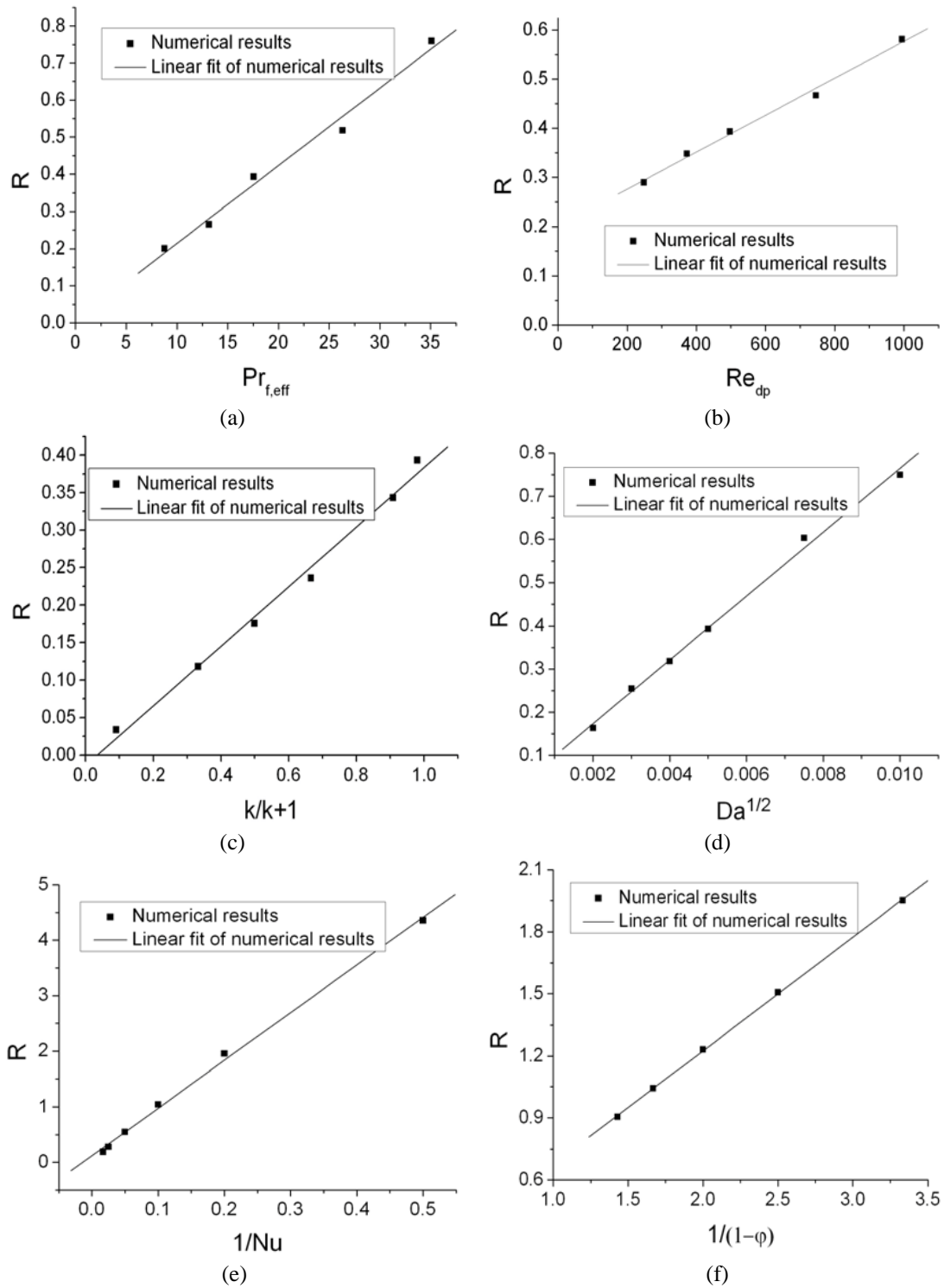


Figure 2. Effects of relevant parameters on R

validation of the assumption of local thermal equilibrium is independent of Reynolds number.

3.4. Comparison with Other Criteria

The criterion derived for the case where conduction is dominant by Carbonell and Whitaker (1984) is expressed as

$$\frac{\varphi (\rho C_p)_f d_p^2}{t} \left(\frac{1}{\lambda_f} + \frac{1}{\lambda_s} \right) \ll 1 \quad (34)$$

where φ , ρ , C_p , d_p , t , λ_f , and λ_s denote porosity, fluid density, fluid specific heat, characteristic length scale of pore size, time scale, fluid conductivity, and solid conductivity, respectively. Then Eq. (34) can be rewritten as

$$\varphi \frac{(\rho C_p)_f d_p^2}{L/V} \frac{1}{\lambda_f} \frac{\kappa + 1}{\kappa} \ll 1 \quad (35)$$

Correspondingly, Eq. (31) can be rewritten as

$$\frac{1}{6B(1-\varphi)} \frac{(\rho C_p)_f d_p^2}{L/V} \frac{1}{\lambda_f} \frac{\kappa}{\kappa + 1} \ll 1 \quad (36)$$

It can be seen that the effect of κ is different in these two criteria. In Eq. (36), if the conductivity of fluid is fixed, the assumption of local thermal equilibrium is valid when κ is decreased. The same conclusion was obtained by Kim et al. (2000). However, the conclusion gained by Eq. (35) is the opposite.

Another criterion, which was derived for the case where convection is the dominant heat-transfer mode, was presented by Kim and Jang (2002) as

$$\text{Pr}_{f,\text{eff}} \text{Re}_{d_p} \text{Da}^{1/2} \frac{\varphi}{\text{Nu}} \ll 1 \quad (37)$$

Compared with Eq. (37), the new criterion defined in Eq. (33) includes the effect of κ and is more comprehensive.

4. CONCLUSION

A general criterion for local thermal equilibrium is presented in terms of parameters including the ef-

fective fluid Prandtl number, the particle Reynolds number, the effective solid-to-fluid thermal conductivity ratio, the Darcy number, the Nusselt number, and porosity. The results presented in this study show that the effect of local thermal equilibrium in porous medium will become larger with the decrease of effective fluid Prandtl number, particle Reynolds number, Darcy number, and the effective solid-to-fluid thermal conductivity ratio, as well as with the increase of the Nusselt number. In addition, when the effective solid-to-fluid thermal conductivity ratio is small, its effect is significant. The effect of Reynolds number is determined by f value. If $0 < f < 1$, the assumption of local thermal equilibrium is valid as the Reynolds number is decreased. If $f > 1$, the assumption of local thermal equilibrium is valid as the Reynolds number is increased.

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