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# Ecological optimization for general heat engines

Rui Long, Wei Liu *

*School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China*

## HIGHLIGHTS

- The heat exchanging processes are non-isothermal.
- The internal dissipations are considered.
- The optimization is conducted under the ecological criterion.
- General upper and lower bounds of the optimal efficiency have been deduced.
- The efficiency bounds of different real heat engines have been proposed.

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## ABSTRACT

We conducted an analysis of efficiency and its bounds for general heat engines under the maximum ecological criterion. For generality, both nonisothermal heat-exchanging processes and internal dissipation were taken into consideration. When the product of the internal dissipation and the heat capacity ratio is one, the efficiency under the maximum ecological criterion is the same as that of the irreversible Carnot model. However, the efficiencies have different physical meanings and optimization spaces. Furthermore, the efficiency is independent of the time it takes to complete each process and the heat conductance. For other situations, numerical calculations were conducted to investigate the parameters' effects on optimal efficiency. When the dimensionless contact times approach zero, the irreversible Carnot model is recovered. The general upper and lower bounds of optimal efficiency are obtained by applying the asymmetric heat capacity ratio limits when the dimensionless contact times approach infinity. In addition, the efficiency of general endoreversible heat engines was investigated. The efficiency bounds of different real-life heat engines under the maximum ecological criterion are proposed.


## 1. Introduction

The optimization of real thermodynamic cycles to save energy and fuel has attracted attention recently. The upper bound of efficiency for heat engines that operate between two heat reservoirs at temperatures $T_c$ and $T_h$ ($T_h > T_c$) is Carnot efficiency [1]. However, reaching Carnot efficiency means vanishing power output and has limited guidance on the practical applications since in Carnot heat engines all processes are quasistatic. The ideal Carnot cycle must be made to go faster to meet real-life demand. Finite-time thermodynamic analysis has provided a way to optimize real heat engines [2–7].

For heat engines, the main optimization criterion is maximum power (MP) output. By considering that the heat transfer processes between the heat reservoirs and the working fluid take a finite amount of time, Curzon and Ahlborn [8] proposed

* Corresponding author.
*E-mail address:* w_liu@hust.edu.cn (W. Liu).

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the concept of the endoreversible Carnot heat engine and derived its efficiency at maximum power output, i.e., the well-known Curzon–Ahlborn (CA) efficiency, $\eta_{CA} = 1 - \sqrt{T_c/T_h}$. The CA model has been modified to describe real-life heat engines more accurately by considering different heat transfer laws between the working medium and the heat reservoirs and the internal dissipation, and some good results with the maximum power output criterion have been obtained [9–15]. Furthermore, taking into account the entropy generation in isothermal processes, which are treated as the inversed functions of process time duration, Esposito et al. [16] proposed the low dissipation model, and obtained the lower and upper bounds of efficiency at MP criterion under asymmetric dissipation limits. In addition, in the low-dissipation model, CA efficiency is reached under symmetric dissipation conditions. Recently, there has been research on the efficiency and its bounds of low-dissipation heat engines with the MP criterion [17–19]. In addition, the efficiency at MP of linear irreversible heat engines described by the Onsager relations and the extended Onsager relations has been studied [20–22].

Real-life heat converter devices may not work at MP output but under a compromise between energy benefits and losses. Angulo-Brown [23] proposed an optimization criterion for Carnot heat engines to account for the benefits and losses of energy, i.e., $E = \dot{W} - T_c\dot{\sigma}$, where $\dot{W}$ is the power output, $T_c$ is the temperature of the cold reservoir, and $\dot{\sigma}$ is the entropy production rate. This became the $\Omega$ criterion for heat engines defined later by Hernández et al. [24]. Based on the $\Omega$ criterion, de Tomas et al. [25] and Long et al. [26] obtained the efficiency limits of heat engines for the low-dissipation model and the minimally nonlinear irreversible model, respectively. However, the lower bounds they proposed were very close to the upper bounds and did not agree with the observed efficiencies. Yan [27] stated that $E = \dot{W} - T_0\dot{\sigma}$, where $T_0$ is the ambient temperature, is more reasonable for heat engines and represents the best compromise between $\dot{W}$ and the power loss $T_0\dot{\sigma}$, which results from entropy generation in the system and its surroundings.

Much research has focused on the irreversible Carnot heat engine under the ecological optimization criterion [2,28–36]. However, there are few reports on nonisothermal heat-exchanging processes. In the heat-exchanging processes in real heat engines, the temperatures of the working medium change continuously to reach the highest or the lowest values, i.e., the heat-exchanging processes are not isothermal. Although both the heat source, heat sink and working fluid can experience temperature variations during a thermodynamic heat-engine cycle [37,38], in this study we focused on the case where the heat source and heat sink temperatures remain constant to identify the effect that the variable temperature of the working fluid has on the efficiency of the cycle. Motivated by Ref. [39], we studied the performance of nonisothermal heat-exchanging processes under the ecological criterion. For generality, the internal dissipation also was included, making our model more realistic. In this paper, we present our systematic study on the efficiency of general heat engines at the maximum ecological criterion and propose its lower and upper bounds. In addition, we discuss the efficiency of general endoreversible heat engines under the ecological figure of merit and the efficiency bounds of different heat engines under the maximum ecological criterion.

## 2. Mathematical model

In a heat engine, a certain amount of heat $Q_h$ is absorbed from the hot reservoir ($T_h$) and some heat $Q_c$ is rejected and goes to the cold reservoir ($T_c$) at the end of a cycle. We assumed that the heat transfer between the heat source and the working medium conforms to Newton's law of cooling [39]:

$$\frac{dQ}{dt} = cm\frac{dT}{dt} = k(T_s - T), \tag{1}$$

where $Q$ is the heat exchanged during the process, $c$ is the heat capacity, $m$ is the mass of the working substance, $T$ is the temperature of the working substance, $T_s$ is the temperature of the heat source, and $k$ is the heat conductance (i.e., contact area multiplied by the heat transfer coefficient). According to Eq. (1), the temperature of the working substance in the heat-absorbing process, $T_{hw}$, is a function of time $t$:

$$T_{hw}(t) = T_h + (T_{h0} - T_h)e^{-t/\Psi_h}, \tag{2}$$

where $T_{h0}$ is the initial temperature of the working medium in the heat-absorbing process and $\Psi_h = c_h m/k_h$ is the temporal response of the working fluid in the heat-absorbing process, where $c_h$ is the specific heat of the working medium and $k_h$ is the heat conductance in the heat-absorbing process. $\Psi_h$ is measured in units of time. The heat absorbed from the hot reservoir is defined by

$$Q_h = \int_0^{\tau_h} k_h(T_h - T_{hw})dt = c_h m(T_h - T_{h0})\left(1 - e^{-\tau_h/\Psi_h}\right), \tag{3}$$

where $\tau_h$ is the time needed to complete the heat-absorbing process. The relative change in the entropy of the working substance in the heat-absorbing process is given by

$$\Delta s_h = \int_0^{\tau_h} \frac{dQ_h}{T} = c_h m \ln\left(\frac{T_h + (T_{h0} - T_h)e^{-\tau_h/\Psi_h}}{T_{h0}}\right). \tag{4}$$

The temperature of the working medium, the heat rejected to the cold reservoir, and the entropy change during the heat-releasing process are given by

$$T_{cw}(t) = T_c - (T_c - T_{c0})\mathrm{e}^{-t/\Psi_c}, \tag{5}$$

$$Q_c = \int_0^{\tau_c} k_c(T_{cw} - T_c)\mathrm{d}t = c_c m(T_{c0} - T_c)(1 - \mathrm{e}^{-\tau_c/\Psi_c}), \tag{6}$$

$$\Delta s_c = \int_0^{\tau_c} \frac{\mathrm{d}Q_c}{T} = -c_c m \ln\left(\frac{T_c - (T_c - T_{c0})\mathrm{e}^{-\tau_c/\Psi_c}}{T_{c0}}\right), \tag{7}$$

where $T_{c0}$ and $c_c$ are the initial temperature and specific heat of the working medium in the heat-releasing process, respectively; $\tau_c$ is the duration of the heat-releasing process; and $\Psi_c = c_c m/k_c$ is the temporal response of the working fluid in the heat-releasing process, where $k_c$ is the heat conductance in the heat-releasing process. $\Psi_c$ is measured in units of time. In this paper, we assumed that the compression and expansion processes proceed instantaneously and the time needed to complete those processes is zero. However, we considered the irreversibility in those two processes. To describe quantitatively the effect of the internal dissipation of the working fluid on the performance of the heat engine, a parameter $I_s = \Delta s_c/\Delta s_h$ has been proposed [12,13]. $I_s$ represents the degree of internal irreversibility resulting from the working fluid. When $I_s = 1$, the heat engine cycle is endoreversible, and when $I_s > 1$, the cycle is internally irreversible. According to Eqs. (4) and (7),

$$\frac{T_c - (T_c - T_{c0})\mathrm{e}^{-\tau_c/\Psi_c}}{T_{c0}}\left[\frac{T_h + (T_{h0} - T_h)\mathrm{e}^{-\tau_h/\Psi_h}}{T_{h0}}\right]^{I_s\alpha} = 1, \tag{8}$$

where $\alpha$ is the ratio of the specific heats of the working medium in the heat-absorbing and -releasing processes. According to Eq. (8), we can rewrite $T_{h0}$ as a function of $T_{c0}$.

Entropy production is defined as

$$\sigma = \frac{Q_c}{T_c} - \frac{Q_h}{T_h}, \tag{9}$$

the ecological function is defined as

$$\dot{E} = \frac{Q_h - Q_c - T_0\sigma}{\tau_h + \tau_c}, \tag{10}$$

and the efficiency is defined as

$$\eta = 1 - \frac{Q_c}{Q_h}. \tag{11}$$

Combining Eqs. (3), (6), and (8) and maximizing Eq. (10) with respect to $T_{c0}$ yield

$$\left(\frac{T_0}{T_h} + 1\right)\frac{(1 - \mathrm{e}^{-\tau_h/\Psi_h})^2}{I_s}\frac{[(1 - \mathrm{e}^{-\tau_c/\Psi_c})\varphi + \mathrm{e}^{-\tau_c/\Psi_c}]^{-1/I_s\alpha - 1}}{\{[(1 - \mathrm{e}^{-\tau_c/\Psi_c})\varphi + \mathrm{e}^{-\tau_c/\Psi_c}]^{-1/I_s\alpha} - \mathrm{e}^{-\tau_h/\Psi_h}\}^2} - \left(\frac{T_0}{T_c} + 1\right)\frac{1 - \eta_C}{\varphi^2} = 0, \tag{12}$$

where $\varphi = T_c/T_{c0}$. Substituting Eqs. (3) and (6) into Eq. (11) yields the efficiency at the maximum ecological criterion:

$$\eta = 1 - \frac{(1 - \eta_C)[(1/\varphi) - 1](1 - \mathrm{e}^{-\tau_c/\Psi_c})}{\alpha\left(1 - \frac{1 - \mathrm{e}^{-\tau_h/\Psi_h}}{[(1 - \mathrm{e}^{-\tau_c/\Psi_c})\varphi + \mathrm{e}^{-\tau_c/\Psi_c}]^{-1/I_s\alpha} - \mathrm{e}^{-\tau_h/\Psi_h}}\right)(1 - \mathrm{e}^{-\tau_h/\Psi_h})}. \tag{13}$$

In general, $\eta$ is derived by solving Eq. (12) for $\varphi$ and substituting the solution into Eq. (13). Below, we discuss systematically the efficiency at the maximum ecological criterion.

## 3. Efficiency when $I_s\alpha = 1$

When $\alpha = 1/I_s$, $I_s \geq 1$; therefore, $\alpha \leq 1$. This means that the specific heat of the working medium in the heat-absorbing process is less than or equal to that of the heat-releasing process. Eqs. (12) and (13) can be rewritten as

$$\left\{\frac{(1 - \mathrm{e}^{-\tau_h/\Psi_h})\varphi}{1 - \mathrm{e}^{-\tau_h/\Psi_h}[(1 - \mathrm{e}^{-\tau_c/\Psi_c})\varphi + \mathrm{e}^{-\tau_c/\Psi_c}]}\right\}^2 = \frac{T_0/T_c + 1}{T_0/T_h + 1} I_s(1 - \eta_C) \tag{14}$$

and

$$\eta = 1 - \frac{1 - \mathrm{e}^{-\tau_h/\Psi_h}[(1 - \mathrm{e}^{-\tau_c/\Psi_c})\varphi + \mathrm{e}^{-\tau_c/\Psi_c}]}{\alpha(1 - \mathrm{e}^{-\tau_h/\Psi_h})\varphi}(1 - \eta_C). \tag{15}$$

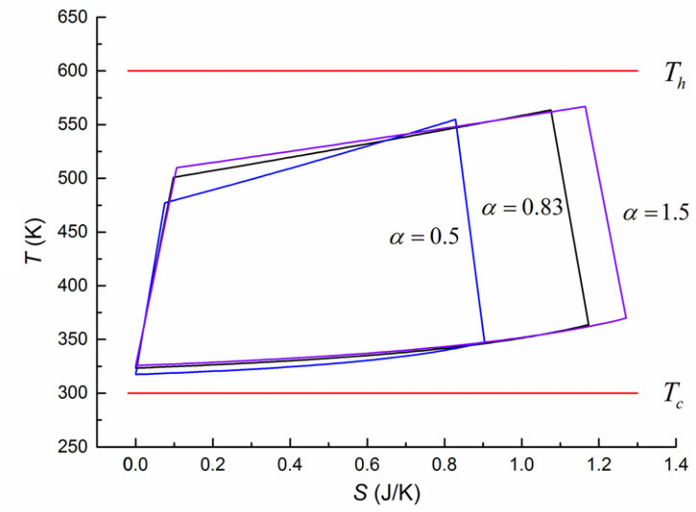


**Fig. 1.** The $T$–$S$ diagram of an optimal heat engine cycle, where $I_s = 1.2$, $\alpha = 0.5, 0.83, 1.5$, $T_h = 600$ K, $T_c = 300\ K$, $\tau_h/\Psi_h = \tau_c/\Psi_c = 1$, and $c_c m = 10$ J/K.

Solving Eq. (14) yields the optimal $\varphi$, which is substituted into Eq. (15) to obtain

$$\eta_E{}^* = 1 - \sqrt{\frac{T_0/T_h + 1}{T_0/T_c + 1} I_s (1 - \eta_C)}. \tag{16}$$

Eq. (16) is independent of the time duration of each process and the heat conductance. The $T$–$S$ diagram of an optimal heat engine cycle is shown in Fig. 1 (the black-lined cycle has $I_s = 1.2$ and $\alpha = 0.83$). The heat-absorbing and -releasing processes are not isothermal. The nonisentropic processes are also presented. When the internal dissipation vanishes, $I_s = \alpha = 1$ and Eq. (16) is reduced to

$$\eta_{E,\mathrm{endo}}{}^* = 1 - \sqrt{\frac{T_0/T_h + 1}{T_0/T_c + 1} (1 - \eta_C)}. \tag{17}$$

This definition of efficiency is the same as that obtained for the endoreversible Carnot model under the ecological criterion [27]. But the physical meanings and optimization spaces are different. In the endoreversible Carnot model, the efficiency with the optimal ecological criterion is obtained by maximizing the ecological function with respect to the time needed for the heat-absorbing and -releasing processes, while in this model, the efficiency is obtained by maximizing the ecological function with respect to the initial temperature of the working medium and the times needed for the processes are treated as constants. Unlike in the endoreversible Carnot heat engine, in this model the temperature of the working medium in either heat-exchanging process is not constant, making it more practical and realistic.

## 4. Efficiency when $I_s\alpha \neq 1$

We defined $\tau/\Psi$ as the dimensionless contact time; it reflects the degree of equilibrium between the temperatures of the working medium and the heat reservoirs. A larger $\tau/\Psi$ means that the working medium is in contact with the heat reservoirs for a longer time, which leads to a higher final temperature in the heat-absorbing process and a lower temperature in the heat-releasing process. When $I_s\alpha \neq 1$, Eq. (12) is transcendental and cannot be solved explicitly. We performed numerical calculations to investigate the effects of $I_s$, $\alpha$, and $\tau/\Psi$ on the optimal efficiencies.

As seen in Figs. 2 and 3, when the dimensionless contact times are the same and equal to 1, the optimal efficiency $\eta_E$ decreases with the increasing internal dissipation parameter $I_s$ and heat capacity ratio $\alpha$. However, when $I_s$ is fixed, the efficiency reaches its upper and lower bounds when $\alpha \to 0$ and $\alpha \to \infty$. Fig. 1 shows the impact of $\alpha$ on the cycle configuration, i.e., a larger $\alpha$ means larger entropy changes in the heat-exchanging processes.

The effect of the dimensionless contact times on the optimal efficiency is illustrated in Figs. 4 and 5. In Fig. 4, when $\alpha < 1/I_s$ and $\tau_c/\Psi_c$ is fixed, the optimal efficiency increases with increasing $\tau_h/\Psi_h$ in a certain interval and reaches its lower and upper bounds under the asymmetric limits $\tau_h/\Psi_h \to 0$ and $\tau_h/\Psi_h \to \infty$, respectively. The lower bound is $\eta_E{}^*$. When $\alpha > 1/I_s$, the optimal efficiency decreases with increasing $\tau_h/\Psi_h$ in a certain interval and reaches its lower and upper bounds under the asymmetric limits $\tau_h/\Psi_h \to \infty$ and $\tau_h/\Psi_h \to 0$, respectively. The upper bound is $\eta_E{}^*$. As mentioned before, when $\alpha = 1/I_s$, the optimal efficiency is $\eta^*$ and is independent of $\tau_h/\Psi_h$.

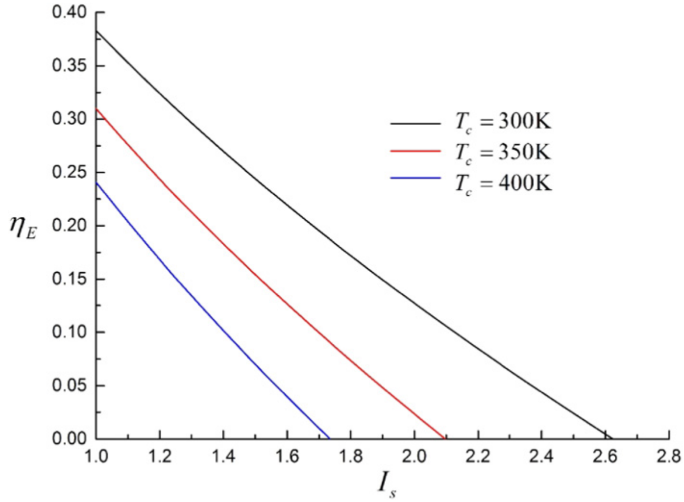


**Fig. 2.** Optimal efficiency as a function of $I_s$ for different cold reservoir temperatures ($T_c = 300, 350, 400$ K), where $T_h = 600$ K, $T_0 = 273$ K, $\alpha = 0.5$, and $\tau_c/\Psi_c = \tau_h/\Psi_h = 1$.

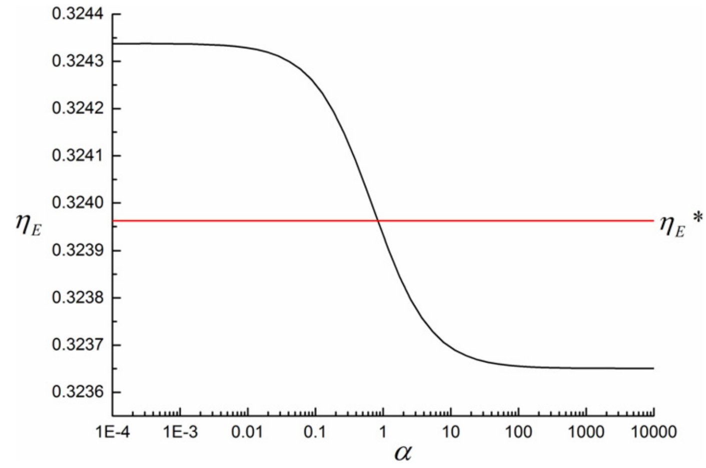


**Fig. 3.** Optimal efficiency as a function of heat capacity ratio, where $T_h = 600$ K, $T_c = 300$ K, $T_0 = 273$ K, $I_s = 1.2$, and $\tau_c/\Psi_c = \tau_h/\Psi_h = 1$.

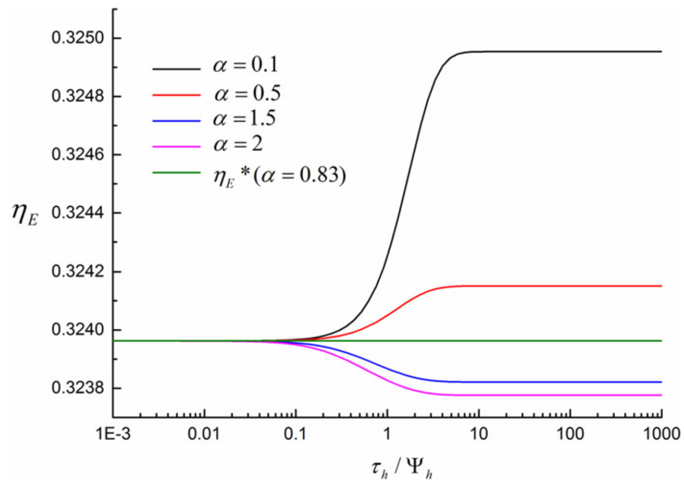


**Fig. 4.** Optimal efficiency as a function of dimensionless contact times in the heat-absorbing process under different heat capacity ratios, where $T_h = 600$ K, $T_c = 300$ K, $T_0 = 273$ K, $I_s = 1.2$, and $\tau_c/\Psi_c = 1$.

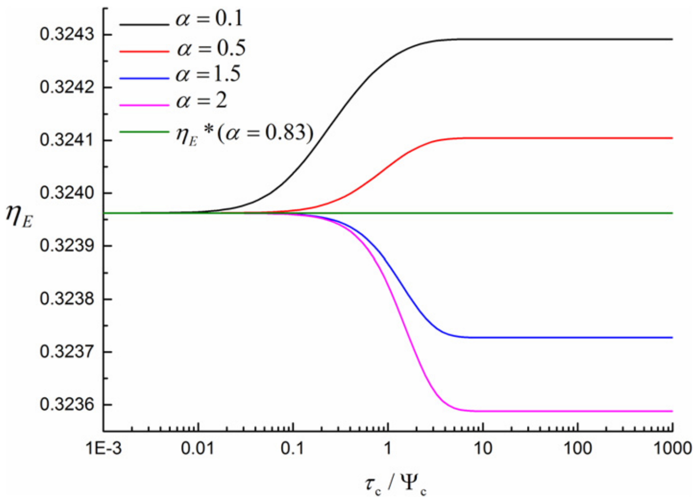


**Fig. 5.** Optimal efficiency as a function of dimensionless contact times in the heat-releasing process under different heat capacity ratios, where $T_h = 600$ K, $T_c = 300$ K, $T_0 = 273$ K, $I_s = 1.2$, and $\tau_h/\Psi_h = 1$.

Fig. 5 shows that when $\alpha < 1/I_s$ and $\tau_h/\Psi_h$ is fixed, the optimal efficiency increases with increasing $\tau_c/\Psi_c$ in a certain interval and reaches its lower and upper bounds when $\tau_c/\Psi_c \to 0$ and $\tau_c/\Psi_c \to \infty$, respectively. The lower bound is $\eta_E{}^*$. When $\alpha > 1/I_s$, the optimal efficiency decreases with increasing $\tau_c/\Psi_c$ in a certain interval and reaches its lower and upper bounds when $\tau_c/\Psi_c \to \infty$ and $\tau_c/\Psi_c \to 0$. The upper bound is $\eta_E{}^*$. As mentioned before, when $\alpha = 1/I_s$, the optimal efficiency is $\eta_E{}^*$ and is independent of $\tau_c/\Psi_c$.

Figs. 4 and 5 show that when $\tau/\Psi \to 0$, the efficiency with the maximum ecological criterion is $\eta_E{}^*$ and is not affected by the heat capacity ratio. Under these conditions, the heat-exchanging processes are short enough so that the final temperature of the working substance is almost equal to its initial temperature after either process. Expanding the term $\exp(-\tau/\Psi)$ to the first order of $\tau/\Psi$ in Eqs. (12) and (13) reduces them to

$$\left\{\varphi\left[1+\frac{(1-\varphi)\tau_c/\Psi_c}{I_s\alpha\tau_h/\Psi_h}\right]^{-1}\right\}^2 = \frac{T_0/T_c+1}{T_0/T_h+1}I_s(1-\eta_C) \tag{18}$$

and

$$\eta = 1-\left[\left(1+\frac{(1-\varphi)\tau_c/\Psi_c}{I_s\alpha\tau_h/\Psi_h}\right)(\varphi/I_s)^{-1}\right](1-\eta_C). \tag{19}$$

Combining Eqs. (18) and (19) yields the same expression as Eq. (16) for $\eta_E{}^*$ and is independent of the heat capacity ratio. When the dimensionless contact times approach 0, the heat-exchanging processes are isothermal and the irreversible Carnot model is recovered, as depicted in Fig. 6(a).

According to the preceding analysis, when $\alpha < 1/I_s$, the optimal efficiency increases with increasing $\tau/\Psi$ and reaches its maximum value when $\tau/\Psi \to \infty$. The lower bound is $\eta_E{}^*$ when $\tau/\Psi \to 0$ and is independent of the heat capacity ratio. When $\alpha > 1/I_s$, the optimal efficiency decreases with increasing $\tau/\Psi$ and reaches its minimum value when $\tau/\Psi \to \infty$. The upper bound is $\eta_E{}^*$ when $\tau/\Psi \to 0$ and is independent of the heat capacity ratio. Thus, the general upper and lower bounds of the optimal efficiency can be obtained when $\tau/\Psi \to \infty$ by applying the asymmetric heat capacity limits $\alpha \to 0$ and $\alpha \to \infty$.

When $\tau/\Psi \to \infty$, the contact time is long enough so that the heat exchange between the working substance and the heat reservoirs is sufficient and the final temperature of the working substance is almost equal to that of the heat reservoir. A typical heat engine cycle is shown in Fig. 6(b). The exponential terms $\exp(-\tau/\Psi)$ in Eqs. (12) and (13) can be eliminated thus simplifying them to

$$\varphi^{1/I_s\alpha+1} = \frac{T_0/T_c+1}{T_0/T_h+1}I_s(1-\eta_C) \tag{20}$$

and

$$\eta = 1-\frac{(1-\varphi)}{\alpha(\varphi-\varphi^{1/I_s\alpha+1})}(1-\eta_C). \tag{21}$$

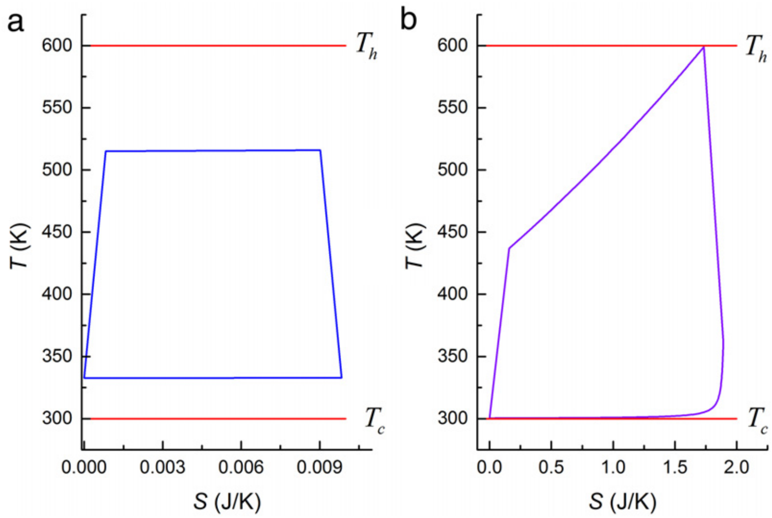


**Fig. 6.** The $T$–$S$ diagrams of two optimal heat engine cycles with different dimensionless contact times: (a) $\tau_h/\Psi_h = \tau_c/\Psi_c = 5$ and (b) $\tau_h/\Psi_h = \tau_c/\Psi_c = 0.01$, where $I_s = 1.2$, $\alpha = 0.5$, $T_h = 600$ K, $T_c = 300$ K, and $c_c m = 10$ J/K.

Combining Eqs. (20) and (21) yields

$$\eta_E = 1 - \left( \frac{1 - \left[ \frac{T_0/T_c+1}{T_0/T_h+1} I_s (1-\eta_C) \right]^{I_s\alpha/(I_s\alpha+1)}}{\alpha \left\{ \left[ \frac{T_0/T_c+1}{T_0/T_h+1} I_s (1-\eta_C) \right]^{I_s\alpha/(I_s\alpha+1)} - \frac{T_0/T_c+1}{T_0/T_h+1} I_s (1-\eta_C) \right\}} \right) (1-\eta_C). \tag{22}$$

By applying the asymmetric limits $\alpha \to 0$ and $\alpha \to \infty$, we get

$$\eta_E{}^{+} = 1 - \frac{I_s(1-\eta_C) \ln\left[ \frac{T_0/T_c+1}{T_0/T_h+1} I_s(1-\eta_C) \right]}{\frac{T_0/T_c+1}{T_0/T_h+1} I_s(1-\eta_C) - 1} \tag{23}$$

and

$$\eta_E{}^{-} = 1 - \frac{\frac{T_0/T_c+1}{T_0/T_h+1} I_s(1-\eta_C) - 1}{\frac{T_0/T_c+1}{T_0/T_h+1} \ln\left[ \frac{T_0/T_c+1}{T_0/T_h+1} I_s(1-\eta_C) \right]}. \tag{24}$$

As mentioned above, $\eta_E{}^{+}$ and $\eta_E{}^{-}$, Eqs. (23) and (24), are the upper and lower bounds of the efficiency of a general heat engine under the maximum ecological criterion.

## 5. General endoreversible heat engines

When internal dissipation is not taken into consideration, the general heat engine is endoreversible, reflecting that the irreversibility results from only the heat exchange between the working medium and the heat reservoirs. Thus, $I_s = 1$ and $\eta_E{}^{*}$ becomes $\eta_{E,\mathrm{endo}}{}^{*}$. According to the above analysis, $\eta_{E,\mathrm{endo}}{}^{*}$ is the lower and upper bounds of the optimal efficiency when $\alpha < 1$ and $\alpha > 1$, respectively. In addition, the optimal efficiency is $\eta_{E,\mathrm{endo}}{}^{*}$ if $\alpha = 1$. Therefore, in endoreversible heat engine cycles such as the Diesel cycle ($c_h = c_p$ and $c_c = c_v$), the Brayton cycle ($c_c = c_h = c_p$), and the Otto cycle ($c_c = c_h = c_v$), where the heat capacity in the heat-absorbing process is not less than that in the heat-releasing process, the maximum efficiency under the maximum ecological criterion is bounded by $\eta_{E,\mathrm{endo}}{}^{*}$, while in cycles such as the Atkinson cycle ($c_h = c_v$ and $c_c = c_p$), where the heat capacity in the heat-absorbing process is less than that in the heat-releasing process, $\eta_{E,\mathrm{endo}}{}^{*}$ is the lower bound.

Furthermore, the general upper and lower bounds of the efficiency can be obtained by applying the asymmetric heat capacity limits $\alpha \to 0$ and $\alpha \to \infty$, respectively. Substituting $I_s = 1$ into Eqs. (23) and (24) yields

$$\eta_{E,\mathrm{endo}}{}^{+} = 1 - \frac{(1-\eta_C) \ln\left[ \frac{T_0/T_c+1}{T_0/T_h+1} (1-\eta_C) \right]}{\frac{T_0/T_c+1}{T_0/T_h+1} (1-\eta_C) - 1} \tag{25}$$

and

$$\eta_{E,\mathrm{endo}}{}^{-} = 1 - \frac{\frac{T_0/T_c+1}{T_0/T_h+1}(1-\eta_C) - 1}{\frac{T_0/T_c+1}{T_0/T_h+1}\ln\left[\frac{T_0/T_c+1}{T_0/T_h+1}(1-\eta_C)\right]}. \tag{26}$$

## 6. Conclusions

We conducted an analysis of efficiency and its bounds with the maximum ecological criterion for general heat engines. For generality, both the nonisothermal heat transfer processes and the internal dissipation were taken into consideration. When $I_s\alpha = 1$, the bounds of the efficiency were found to be $\eta_E{}^*$ and independent of the time it takes to complete each process and of the heat conductance. When $I_s\alpha \neq 1$, we conducted numerical calculations to investigate the effect of the parameters $I_s$, $\alpha$, and $\tau/\Psi$ on the optimal efficiency under the ecological criterion. We found that the optimal efficiency decreases monotonously with the increase in $I_s$ and $\alpha$. When $\tau/\Psi \to 0$, the irreversible Carnot model is reached and the efficiency at the maximum ecological criterion is $\eta_E{}^*$ and is independent of $\alpha$. When $\tau/\Psi \to \infty$, the general upper and lower bounds of the optimal efficiency are obtained by applying the asymmetric heat capacity limits $\alpha \to 0$ and $\alpha \to \infty$, respectively.

In addition, we studied the efficiency of general endoreversible heat engines with the maximum ecological criterion and analyzed the efficiency bounds of different heat engines. In heat engine cycles such as the Brayton, the Otto, and the Diesel, where the heat capacity in the heat-absorbing process is not less than that in the heat-releasing process, $\eta_{E,\mathrm{endo}}{}^*$ is the upper bound of the efficiency, and in cycles such as the Atkinson, where the heat capacity in the heat-absorbing process is less than that in the heat-releasing process, $\eta_{E,\mathrm{endo}}{}^*$ is the lower bound of the efficiency. We also derived the general upper and lower bounds of the efficiency of general endoreversible heat engines. This study may provide practical insight to help in the design and operation of real heat engines.

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