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ANALYSIS OF FIELD SYNERGY ON NATURAL CONVECTIVE HEAT TRANSFER IN POROUS MEDIA

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ABSTRACT

Based on the mathematical model to describe simultaneous heat, moisture and gas migration in porous media, the idea of enhancing convective heat transfer by the synergy between velocity and temperature gradient is applied to derive the expressions about natural convective heat transfer in unsaturated porous media. The final expressions are written as two formulas for the overall and local heat transfer coefficients between the fluid and the wall. For the rectangular porous enclosure with the length-to-width aspect ratio of three, the phenomenon of field synergy on natural convection in unsaturated porous media is observed and verified through numerical calculations, which indicates that the strength of natural convection in porous media is determined not only by temperature difference, fluid velocity and fluid properties, but also by the synergy between fluid velocity and temperature gradient. © 2003 Elsevier Science Ltd

Introduction

Starting from energy equation of single-phase convection, Guo *etc.* [1,2,3] gave a second look at the mechanism of convective heat transfer, in which heat convection was analogical with heat conduction with heat sources. It was presented that the strength of convective heat transfer is largely determined by the heat source that is related not only to fluid properties, temperature difference and fluid velocity, but also to the synergy between fluid velocity and temperature gradient. By the idea of field synergy, fluid motion may enhance or reduce heat transfer or have no contribution to heat transfer. The reduction of the intersection angle between fluid velocity and temperature gradient can effectively enhance convective heat transfer. The principle of field synergy illuminates how local behaviors of convective heat transfer affect the global heat transfer performance. With this concept, the mechanism of convective heat transfer enhancement can be understood quite well.

Tao [4] extended the idea of field synergy from parabolic to elliptic model for fluid flow and heat transfer. Some numerical and practical examples were provided to show the correctness and the importance of this newly developed concept. Zhao [5] conducted experimental study on convective heat transfer in porous media saturated with water. The results showed that when the intersection angle between fluid velocity and temperature gradient is equal to zero, the linear dependence between Nusselt number and Peclet number, namely $Nu=Pe$, could be obtained for the small Peclet numbers. Obviously, such heat convection is far more efficient compared with the case of the intersection angle 90° for $Nu=1.329Pe^{1/2}$ [6].

The concept of field synergy has not hitherto been applied and verified in heat transport through unsaturated porous media. Heat and moisture transfers in unsaturated porous media are coupled in a very complicated way. In view of the complexity of multi-phase flow and phase change, Slattary [7] and Whitaker [8] originally developed the volume averaging method to solve the problem. Based on the thermodynamics theory, Philip and DeVries [9], DeVries [10,11] previously established a two-field model to describe combined heat and moisture transfer in porous media. Referring to above approaches, Liu *etc.* [12,13] developed a seven-field mathematical model to describe the simultaneous heat and mass transfer with phase change in porous media. Consider the porous media that have relatively higher porosities. Under the assumption of continuous gas phase, when gas is heated, natural convection can develop in the pore space due to buoyancy force. Natural convective heat transfer in a tilted rectangular enclosure packed with unsaturated porous material was analyzed numerically in the reference [14], and the phenomenon of synergy between fluid velocity and temperature gradient was observed in the vicinity of wall.

In this paper, the idea of field synergy is applied to derive the expressions of natural convective heat transfer in unsaturated porous media. From the point of field synergy, the mechanism of enhancing natural convective heat transfer in unsaturated porous media could be understood more clearly. In addition, a numerical example is provided to verify the validity of the expressions.

Model of Heat and Mass Transfer in Unsaturated Porous Media

According to volume averaging method, a macroscopic variable is defined as an appropriate mean over a sufficiently large representative elementary volume (REV). The local REV is chosen such that it is the smallest differential volume that results in statistically meaningful local average properties. Thus, it is requested that the length scale of REV (*l*) is much larger than the pore scale (*d*), but considerably smaller than the length scale of the macroscopic flow domain (*L*), i.e. $d<<l<<L$.

By means of general conservation for continuum mechanics, a mathematical model [12] has been established to describe the combined heat and mass transfer in unsaturated porous media. In the present model, the local thermodynamics equilibrium is assumed among solid, liquid and gas phases. Combination of water vapor and air are regarded as gaseous mixture. Besides the bulk motion of gaseous mixture, the vapor diffusion motion is also taken into account. Under the assumption of homogeneous, isotropic and regular porous media with continuous gaseous phase, relatively strong gaseous natural convection can develop in the porous enclosure with higher porosity due to buoyancy force caused by temperature difference. This is to say the bulk flow of gaseous mixture can form regularly. In addition, the principle of partial pressure law is introduced in the mathematical modeling.

Continuity Equations

Liquid: $$\nabla\cdot(\rho_l\varepsilon_l\vec{V}_l) = -\dot{m} \tag{1}$$

Vapor: $$\nabla\cdot[\rho_v\varepsilon_g(\vec{V}_v+\vec{V}_g)] = \dot{m} \tag{2}$$

Air: $$\nabla\cdot(\rho_a\varepsilon_g\vec{V}_g) = 0 \tag{3}$$

Momentum Equations

Liquid phase: $$\varepsilon_l(\vec{V}_l\cdot\nabla)(\rho_l\vec{V}_l) - \dot{m}\vec{V}_l = -\rho_l g D_l/K_l\nabla\varepsilon_l - \rho_l\varepsilon_l\vec{g} - \rho_l\varepsilon_l g/K_l\vec{V}_l \tag{4}$$

Gas phase: $$\varepsilon_g(\vec{V}_g\cdot\nabla)(\rho_g\vec{V}_g) + \dot{m}\vec{V}_g = -\nabla P + \mu_g\nabla^2\vec{V}_g - \rho_g\varepsilon_g\vec{g} - \rho_g\varepsilon_g g/K_g\vec{V}_g \tag{5}$$

Vapor Diffusion Equation

$$\vec{V}_v = -D_{va}\nabla\rho_v / \rho_v \tag{6}$$

Energy Equation

$$\varepsilon_l\vec{V}_l \cdot \nabla(\rho_l C_l T) + \varepsilon_g\vec{V}_g \cdot \nabla(\rho_g C_g T) = \nabla\cdot(K_m\nabla T) - \dot{m}\gamma \tag{7}$$

Where $\vec{V}_g$ is fluid phase-averaged velocity that represents the orientation of fluid flow in the REV, which can well show the trend of gaseous motion in porous media. Based on the relation between fluid phase-averaged velocity and temperature gradient, we could analyze the mechanism of enhancing convective heat transfer in porous media.

Derivation of Expressions According to Field Synergy Principle

In the energy equation, the mechanism of enhancing natural convective heat transfer in unsaturated porous media is analyzed from the view of field synergy. By means of gaseous continuity equation and vapor diffusion equation, the source term of phase change can be incorporated into the $V\cdot\nabla T$ term and the $\nabla\cdot(\nabla T)$ term of energy equation. Then we integrate this energy equation over the thermal boundary layer. In terms of the Guass theorem, the volume integral can be rewritten as two area integrals along the interface of thermal boundary layer and along the wall, respectively. Finally, according to the definition of the thermal boundary layer, two expressions of the overall and local heat transfer coefficients through the wall are obtained. The detailed derivation is as follows.

For simplification, it is assumed that the density of air is uniform. Hence equation (3) can be rewritten as

$$\nabla\cdot(\varepsilon_g\vec{V}_g) = 0\,. \tag{8}$$

If we suppose that the density of vapor is only dependent on temperature, the vapor diffusion equation is

$$\vec{V}_v = -D_{va}\frac{1}{\rho_v}\frac{d\rho_v}{dT}\nabla T\,. \tag{9}$$

Substituting equation (8) and equation (9) into equation (2) yields

$$\dot{m} = -\nabla \cdot (\varepsilon_g D_{va} \frac{d\rho_v}{dT} \nabla T) + \varepsilon_g \frac{d\rho_v}{dT} \vec{V}_g \cdot \nabla T . \tag{10}$$

In continuous pore space, when the natural convection of gas phase is dominant, energy equation reduces to

$$\varepsilon_g \rho_g c_g (\vec{V}_g \cdot \nabla T) = \nabla \cdot (K_m \nabla T) - \dot{m}\gamma . \tag{11}$$

Substituting equation (10) into equation (11), we get

$$(\varepsilon_g \rho_g c_g + \varepsilon_g \gamma \frac{d\rho_v}{dT}) \vec{V}_g \cdot \nabla T = \nabla \cdot [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T] . \tag{12}$$

Consider the convective heat transfer in unsaturated porous media near the wall as shown in FIG.1 (a). *A* represents a continuous and differentiable interface. *a* and *b* are two positions where the interface *A* intersects with the wall. And Ω is the closed domain. Integrating equation (12) over the domain Ω, we obtain

$$\int_\Omega [(\varepsilon_g \rho_g c_g + \varepsilon_g \gamma \frac{d\rho_v}{dT}) \vec{V}_g \cdot \nabla T] d\Omega = \int_\Omega \{\nabla \cdot [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T]\} d\Omega . \tag{13}$$

FIG.1
Integral domains in unsaturated porous media

By using the Gauss theorem to reduce the integral dimension, the right-hand side of equation (13) can be extended as

$$\int_\Omega \{\nabla \cdot [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T]\} d\Omega = \int_A [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T] dS + \int_{S_{ab}} [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T] dS .$$

Then equation (13) becomes

$$\int_{S_{ab}} [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T] dS = \int_\Omega [(\varepsilon_g \rho_g c_g + \varepsilon_g \gamma \frac{d\rho_v}{dT}) \vec{V}_g \cdot \nabla T] d\Omega - \int_A [(K_m + \varepsilon_g \gamma D_{va} \frac{d\rho_v}{dT}) \nabla T] dS . \tag{14}$$

Due to the temperature difference between gaseous phase and the wall, a thermal boundary layer may be defined near the wall. Consider the geometry of thermal boundary layer δ (δ << length of arc *ab*), as shown in FIG.1 (b). If the integral domain is replaced by the entire thermal boundary layer, namely

$\Omega=\Omega_\delta$ and $A=A_\delta$, equation (14) is now written in the form of

$$\int_{S_{ab}}[(K_m+\varepsilon_g\gamma D_{va}\frac{d\rho_g}{dT})\nabla T]dS=\int_{\Omega_\delta}[(\varepsilon_g\rho_g c_g+\varepsilon_g\gamma\frac{d\rho_v}{dT})\vec{V}_g\cdot\nabla T]d\Omega-\int_{A_\delta}[(K_m+\varepsilon_g\gamma D_{va}\frac{d\rho_v}{dT})\nabla T]dS. \quad (15)$$

According to the definition of thermal boundary layer, we have $\int_{A_\delta}[(K_m+\varepsilon_g\gamma D_{va}\frac{d\rho_v}{dT})\nabla T]dS=0$.

So equation (15) reduces to

$$\int_{S_{ab}}[(K_m+\varepsilon_g\gamma D_{va}\frac{d\rho_v}{dT})\nabla T]dS=\int_{\Omega_\delta}[(\varepsilon_g\rho_g c_g+\varepsilon_g\gamma\frac{d\rho_v}{dT})\vec{V}_g\cdot\nabla T]d\Omega. \quad (16)$$

Introduce the following dimensionless variables

$$\tilde{\rho}_v=\frac{\rho_v}{\rho_g},\theta=\frac{T-T_0}{\Delta T},\tilde{\vec{V}}_g=\frac{\vec{V}_g\delta}{a_m},X=\frac{x}{\delta},Y=\frac{y}{\delta},\text{ where } a_m=\frac{\dot{K}_m}{\rho_g c_g}.$$

The corresponding dimensionless form of equation (16) is given by

$$\int_{S_{ab}}[(1+\varepsilon_g\frac{\gamma}{c_g\Delta T}\frac{D_{va}}{a_m}\frac{d\tilde{\rho}_v}{d\theta})\nabla\theta]dS=\int_{\Omega_\delta}[(1+\frac{\gamma}{c_g\Delta T}\frac{d\tilde{\rho}_v}{d\theta})\varepsilon_g\tilde{\vec{V}}_g\cdot\nabla\theta]d\Omega.$$

Defining two dimensionless numbers $Ja=\frac{\gamma}{c_g\Delta T}, Le=\frac{a_m}{D_{va}}$, we have

$$(1+\varepsilon_g\frac{Ja}{Le}\frac{d\tilde{\rho}_v}{d\theta})\int_{S_{ab}}(\nabla\theta)dS=(1+Ja\frac{d\tilde{\rho}_v}{d\theta})\int_{\Omega_\delta}\varepsilon_g(\tilde{\vec{V}}_g\cdot\nabla\theta)d\Omega.$$

The overall Nusselt number is then obtained as

$$Nu_m=\int_{S_{ab}}(\nabla\theta)dS=\frac{(1+Ja\frac{d\tilde{\rho}_v}{d\theta})}{(1+\varepsilon_g\frac{Ja}{Le}\frac{d\tilde{\rho}_v}{d\theta})}\int_{\Omega_\delta}\varepsilon_g(\tilde{\vec{V}}_g\cdot\nabla\theta)d\Omega\cdot \quad (17)$$

Differentiating equation (17) along the X-direction, we obtain the corresponding expression of the local Nusselt number in two-dimension

$$Nu_x=\frac{(1+Ja\frac{d\tilde{\rho}_v}{d\theta})}{(1+\varepsilon_g\frac{Ja}{Le}\frac{d\tilde{\rho}_v}{d\theta})}\int_0^1\varepsilon_g(\tilde{\vec{V}}_g\cdot\nabla\theta)dY. \quad (18)$$

The vector dot product $\tilde{\vec{V}}_g\cdot\nabla\theta$ in equation (17) and equation (18) can be expressed as

$\tilde{\vec{V}}_g \cdot \nabla\theta = \left|\tilde{\vec{V}}_g\right| \left|\nabla\theta\right| \cos\beta$. Here β is the intersection angle between fluid velocity and temperature gradient.

From equation (17) and equation (18), we can see that for the natural convection in unsaturated porous media, the Nusselt numbers depend intensively on gas-phase velocity, temperature difference and fluid properties. However, they also depend on the synergy between gas-phase velocity and temperature gradient. The intersection angle between gas-phase velocity and temperature gradient plays an important role in both the overall and the local heat transfers. Equation (17) indicates that the increase in magnitudes of gas-phase velocity and temperature gradient and mean value of $cos\beta$ can enhance the overall heat transfer. For the local heat transfer, equation (18) implies that the smaller the intersection angle is, the greater the heat transfer rate becomes for β<90°.

Numerical Verification of Field Synergy

We exemplify natural convection in a rectangular enclosure embedded in unsaturated porous media to numerically verify the expressions derived from field synergy principle. Consider the rectangular enclosure as depicted in FIG.2, where T_1 and T_0 represent the temperature of the hot and the cold walls respectively, while the other two walls are adiabatic.

The enclosure is packed with sand and the working fluid is water. In the present investigation, we just analyze the steady natural convective heat transfer with phase change in unsaturated porous enclosure. Concerning the porous enclosure with length-to-width aspect ratio of three, when the Darcy-Rayleigh number R=$DaRa$=150, we numerically simulate its isotherms, streamlines and contours of evaporation and condensation rates as shown. In FIG.3 (a-b), one may see that the flow is fully developed in the enclosure. FIG.3 (c) shows that there are a core of evaporation and a core of condensation appearing at lower- and upper-most corners.

Based on the local heat transport behavior, the phenomenon of synergy between fluid velocity and temperature gradient can be observed. For the porous media near the right wall of the enclosure (see FIG.2), the variations of the product of magnitudes of gas-phase velocity and temperature gradient vectors $\left|\tilde{\vec{V}}_g\right| \cdot \left|\nabla\theta\right|$, the intersection angle between those two vectors β, and the local Nusselt number Nu_x in the X-direction are shown in FIG.4. We arbitrarily choose two points C and D along the cross-section of the

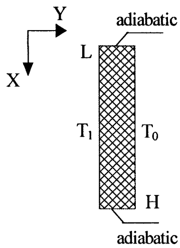
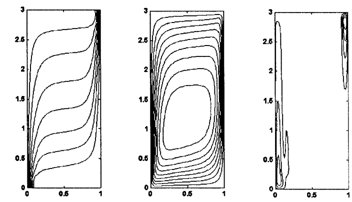
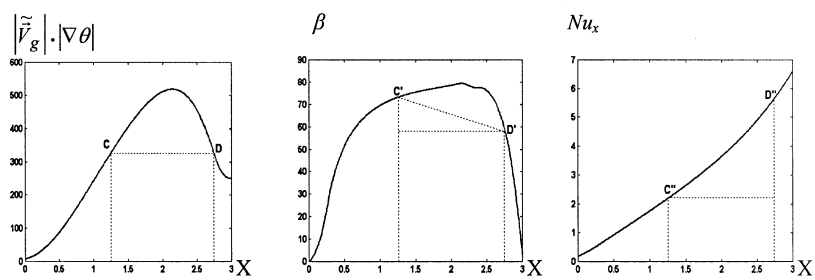


FIG.2
Physical model



(a) (b) (c)

FIG.3
(a) isotherms, (b) streamlines and (c) contours of evaporation and condensation rates for R=150



(a) (b) (c)

FIG.4
Variations of $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$, β and Nu_x with X for R=150

porous enclosure near the right wall to let $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$ (C) = $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$ (D) shown in FIG.4 (a). There should be the counterparts for C and D in FIG.4 (b)(c), namely C' and C" corresponding to C, D' and D" to D. Their geometry positions meet X(C')=X(C")=X(C) and X(D')=X(D")=X(D). It can be found that when the product $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$ (C) = $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$ (D), the intersection angle β (C')>β (D'), but the local Nusselt number Nu_x (C")<Nu_x (D"). It can be seen that the local Nusselt number do depend not only on the magnitudes of vectors $\tilde{\vec{V}}_g$ and $\nabla\theta$, but also on their intersection angle. When the product $\left|\tilde{\vec{V}}_g\right|\cdot|\nabla\theta|$ is uniform, the smaller is the intersection angle, the greater the heat transport rate becomes for β<90°.

Conclusions

The expressions derived from the idea of field synergy facilitate the analysis on the mechanism of enhancing convective heat transfer in porous media. Through numerical calculation for the rectangular porous enclosure, we verify that the strength of natural convection in unsaturated porous media is determined not only by temperature difference, gas-phase velocity and fluid properties, but also by the field synergy between gas-phase velocity and temperature gradient. When the product of magnitudes of velocity and temperature gradient vectors is uniform, the smaller is the intersection angle, the greater the heat transport rate becomes for β<90°. Therefore the strength of natural convective heat transfer in unsaturated porous media can be changed by the synergy between velocity and temperature gradient.

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Nomenclature

a_m	thermal diffusivity of porous media, m^2/s
c	specific heat, $J/(Kg{\cdot}K)$
D_l	diffusivity of liquid in porous medium, m^2/s
D_{va}	molecular diffusivity of vapor in air, m^2/s
g	acceleration of gravity, m/s^2
H	height of the porous enclosure, m
J_a	factor of phase change, dimensionless number
K_g	infiltrating conductivity of gas-mixture, m/s
K_l	hydraulic conductivity of liquid, m/s
K_m	apparent thermal conductivity, $W/(m{\cdot}K)$
L	width of the porous enclosure, m
Le	Lewis number
$\dot{m}$	mass rate of phase change, $Kg/(m^3{\cdot}s)$
Nu_m	global Nusselt number
Nu_x	local Nusselt number
P	pressure, Pa
T	temperature, K(℃)
$\vec{V}$	fluid phase-averaged velocity vector, m/s

Greek Symbols

γ	latent heat, J/Kg
ε	phase content, %
θ	dimensionless temperature
μ	viscosity, $Kg/(m{\cdot}s)$

ρ	density, Kg/m^3

Subscripts

a	air
g	gas phase
l	liquid phase
m	apparent mean, mass
v	vapor

Superscripts

$\sim$	dimensionless quantities

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