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Coefficient of performance and its bounds for general refrigerators with nonisothermal processes

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Abstract

An analysis of the coefficient of performance (COP) and its bounds at maximum χ figure of merit for refrigerators is conducted and the heat transfer processes are described using Newton's law of cooling with a time-dependent heat conductance. The upper bound of the COP, $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, is independent of the time duration completing either process or the heat conductance. The optimal temperature profiles in the heat exchanging processes are analyzed under different dimensionless contact time limits. Provided the dimensionless contact times satisfy certain relations, the endoreversible model is recovered. Furthermore, numerical calculations are conducted to investigate the effect of different heat conductances on temperature profiles under the maximum χ criterion. This work can guide design and operation of actual refrigerators.

Keywords: refrigerators, heat conductance, coefficient of performance
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(Some figures may appear in colour only in the online journal)

1. Introduction

Conditioned on energy saving and fuel depletion, the optimization of real thermodynamic cycles has attracted increasing attention. Refrigerators are used widely in our everyday life, however, refrigerators cannot achieve the Carnot coefficient of performance (COP). In the ideal Carnot refrigerator cycle, all processes are quasistatic and infinitely slow, which leads to a vanishing cooling load rate. Therefore, the ideal Carnot refrigerator must be sped up to meet

the actual demand. Finite time thermodynamic analysis has provided a perspective of optimization for real refrigerators.

The maximum power output is often adopted as the criterion for optimizing real heat engines. Considering only irreversibility caused by heat transfer between the heat reservoirs and working substance during the isothermal processes, Curzon–Ahlborn (CA) [1] proposed the endoreversible Carnot heat engine and deduced its efficiency at maximum power output. That is the groundbreaking CA efficiency. Later, Esposito *et al* [2] proposed the low dissipation model by considering the entropy generation in isothermal processes, which are treated as an inversed function of the process time duration in the heat absorbing and releasing processes, respectively. Then, they obtained the lower and upper bounds of the efficiency at maximum power output. Furthermore, the CA efficiency is recovered under the symmetric dissipation condition. Considering the temperature changes in the heat absorbing and releasing processes and the fact that the heat transfer obey Newton's law of cooling, Yan and Guo [3] obtained the upper and lower bounds of efficiency at maximum power output.

However, for refrigerators, the minimum power input is not an appropriate optimization criterion [4]; therefore, much research has focused on selecting figures of merit to optimize refrigerators. By maximizing the per-unit-time COP, Velasco *et al* [5] obtained the upper bound of COP, $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, i.e., the CA coefficient of performance, for endoreversible refrigerators, in which $\varepsilon_C = T_c/(T_h - T_c)$ is the Carnot COP, where T_c and T_h are the temperatures of the cold and hot reservoirs, respectively. Hernandez *et al* [6] proposed the Ω figure of merit, which indicates a compromise between energy benefits and losses. Later, further efforts are dedicated to the optimization using the Ω criterion. De Tomas *et al* [7] and Hernandez *et al* [8] investigated the COP of refrigerators based on the Ω figure of merit and proposed the upper and lower bounds of the COP. The same bounds have also been obtained through the minimally nonlinear irreversible refrigerator model under the tight coupling condition [9]. Furthermore, under the maximum cooling power criterion, Apertet *et al* [10] studied endoreversible and exoreversible refrigerators and claimed that the real-life working conditions of the refrigerators do not correspond to a maximum cooling power, but rather to a tradeoff between cooling power and cooling efficiency. In addition, Yan and Chen [11] conducted the optimization with the target function $\varepsilon\dot{Q}_c$, where $\dot{Q}_c$ is the cooling load rate of the refrigerators. Further, de Tomas *et al* [12] introduced the unified optimization criterion χ both for heat engines and refrigerators. This criterion is defined as the efficiency, or COP multiplied by the heat absorbed Q_{in}, divided by the cycle time duration t_{cycle}, i.e., $\chi = zQ_{in}/t_{cycle}$, where z is the efficiency for heat engines or COP for refrigerators. By taking χ as the target function, based on the low dissipation model, Wang *et al* [4] proposed that the COP at maximum χ was bounded between 0 and $\left(\sqrt{9 + 8\varepsilon_C} - 3\right)/2$. Moreover, through the minimally nonlinear irreversible refrigerator model, Izumida *et al* [13] obtained the same bounds as those in [4] under the tight coupling condition. However, the arbitrariness in selecting the thermal flux may result in fatal consequences of the optimization for refrigerators. Thus, Sheng and Tu considered a more general model with weighted reciprocal of temperature and thermal flux [14, 15]. In addition, Allahverdyan *et al* [16] investigated quantum refrigerators and obtained new bounds of COP under the χ figure of merit.

Thus far, in studying the COP of real refrigerators, few studies have considered the temperature changes of the working substance during the heat absorbing and releasing processes. For an actual heat transfer process, such as the development of the heat transfer, the temperature difference between the working medium and the heat reservoir decreases. The heat conductance cannot be held constant because it should be affected by the temperature difference. In this paper, we study a more general and realistic refrigerator cycle model that accounts for the temperature changes of the working substance and the time-dependent heat

conductance in the heat exchanging processes. Then, the COP bounds at the maximum χ criterion are derived. In addition, the optimal temperature profiles in the heat exchanging processes are analyzed under different dimensionless contact time limits. Finally, conclusions are drawn.

2. Mathematical model

In refrigerators, the cooling load Q_c is absorbed from the cold reservoir (T_c), and a certain amount of heat Q_h, is evacuated to the hot reservoir (T_h) at the end of a cycle. The heat transfer law between the heat source and the working substance is assumed to conform to Newton's law of cooling

$$\frac{dQ}{dt} = cm\frac{dT}{dt} = k\left(T_s - T\right) \tag{1}$$

where c is the heat capacity, m is the working substance mass, T is the working substance temperature, T_s is heat source temperature, and k is the heat conductance (contact area multiplied by the heat transfer coefficient). Constant heat conductances have usually been a prerequisite for analytically studying heat devices [5, 17, 18]. Actually, as the heat transfer process develops, the temperature difference between the working medium and the heat reservoir decreases. Therefore, the heat conductance should not be kept constant, but rather should be a time-dependent decreasing variable. In this sense, we assume the heat conductance between the working medium and the cold and hot reservoirs conforms the following power law relations, $k_c = a\left(\frac{T_c - T}{T_c - T_{c0}}\right)^n$, $k_h = b\left(\frac{T - T_h}{T_{h0} - T_h}\right)^n$, $n > 0$, where the subscripts c and h indicate the cold and hot reservoir, respectively, and a and b are the initial heat conductances. The initial temperature of the working substance is T_{c0} and T_{h0}, at the beginning of the cooling and heating processes, respectively. In either heat exchanging process, T is a function of time t, so is the proposed heat conductance. When n approaches zero, the heat conductance is held constant during the heat exchanging processes, meaning the heat transfer processes follow normal linear law with constant heat conductance; the present model is recovered to that found in [3]. Therefore, the model with time-dependent conductance studied in this paper should be more general. According to equation (1), the working substance temperature in the heat absorbing process is a function of time t

$$T_{cw}(t) = T_c - \left(T_c - T_{c0}\right)\left(nt/\Psi_c + 1\right)^{-1/n} \tag{2}$$

where $\Psi_c = cm/a$, which reflects the temperature increase degree of the working medium in the time absorbing process and has the dimension of time. The time duration of the heat absorbing process is denoted as τ_c and the cooling load can be calculated as

$$Q_c = \int k_c\left(T_c - T_{cw}\right)dt = cm\left(T_c - T_{c0}\right)\left[1 - \left(n\tau_c/\Psi_c + 1\right)^{-1/n}\right] \tag{3}$$

The relative entropy change of the working substance in the heat absorbing process is given by

$$\Delta s_c = \int \frac{dQ_c}{T} = cm\ln\frac{T_c - \left(T_c - T_{c0}\right)\left(n\tau_c/\Psi_c + 1\right)^{-1/n}}{T_{c0}} \tag{4}$$

Similarly, the heat evacuated to the hot reservoir and the entropy change during the heat releasing process are given by

$$T_{hw}(t) = T_h + \left(T_{h0} - T_h\right)\left(n\tau_h/\Psi_h + 1\right)^{-1/\mathrm{n}} \tag{5}$$

$$Q_h = cm\left(T_{h0} - T_h\right)\left[1 - \left(n\tau_h/\Psi_h + 1\right)^{-1/\mathrm{n}}\right] \tag{6}$$

$$\Delta s_h = -cm\ln\frac{T_h + \left(T_{h0} - T_h\right)\left(n\tau_h/\Psi_h + 1\right)^{-1/\mathrm{n}}}{T_{h0}} \tag{7}$$

where $\Psi_h = cm/b$, which reflects the temperature increase degree of the working medium in the time releasing process and has the dimension of time. Here, τ_h is the time duration of that process. In this paper, we assume that the compressing and expanding processes are isentropic and the time for completing those processes is zero. After a cycle is completed, the working substance returns to its initial state and the total entropy change of the working substance should be zero, i.e., $\Delta s_h + \Delta s_c = 0$. Then, we have

$$\frac{T_c - \left(T_c - T_{c0}\right)\left(n\tau_c/\Psi_c + 1\right)^{-1/\mathrm{n}}}{T_{c0}}\frac{T_h + \left(T_{h0} - T_h\right)\left(n\tau_h/\Psi_h + 1\right)^{-1/\mathrm{n}}}{T_{h0}} = 1 \tag{8}$$

The coefficient of performance ε is

$$\varepsilon = \frac{Q_c}{Q_h - Q_c} \tag{9}$$

The figure of merit $\chi = \varepsilon Q_c/t_{cycle}$ is adopted as the criterion here, which can be expressed as

$$\chi = \frac{Q_c^2}{\left(Q_h - Q_c\right)\left(\tau_h + \tau_c\right)} \tag{10}$$

Combining equations (8) and (10), and maximizing χ with respect to T_{c0}, we obtain

$$M^2 - 2M + \frac{\varepsilon_{\mathrm{C}}}{\varepsilon_{\mathrm{C}} + 1} = 0 \tag{11}$$

where

$$M = \frac{\left[1 - \left(\frac{n\tau_h}{\Psi_{\mathrm{h}}} + 1\right)^{-1/\mathrm{n}}\right]\varphi}{1 - \left(\frac{n\tau_h}{\Psi_{\mathrm{h}}} + 1\right)^{-1/\mathrm{n}}\left\{\left[1 - \left(\frac{n\tau_c}{\Psi_{\mathrm{c}}} + 1\right)^{-1/\mathrm{n}}\right]\varphi + \left(\frac{n\tau_c}{\Psi_{\mathrm{c}}} + 1\right)^{-1/\mathrm{n}}\right\}} \tag{12}$$

where $\varphi = T_c/T_{c0}$. In the heat absorbing process, the initial temperature of the working medium should be lower than that of the cold reservoir, $\varphi \geqslant 1$. Meanwhile, substituting equations (3) and (6) into equation (9), the COP of the refrigerator can be rewritten as

$$\varepsilon = \frac{2}{\frac{\varepsilon_{\mathrm{C}} + 1}{\varepsilon_{\mathrm{C}}}M^2 - 1} \tag{13}$$

Combining equations (11)–(13), we obtain

$$\varepsilon^{\max\chi} = \sqrt{\varepsilon_C + 1} - 1 \tag{14}$$

Equation (14) is independent of the time duration in either process and is equal to the CA coefficient of performance, which has been obtained using the endoreversible refrigerator model [11]. Although the upper bounds of the COP for those two models are the same, they have different physical meanings and optimization spaces. In the endoreversible model, the upper bound of the COP is obtained by maximizing χ with respect to the time durations of the heat absorbing and releasing processes, respectively. In this model, the upper bound is obtained by maximizing χ with respect to the initial temperature of the working medium and the time durations are treated as constants. Unlike Carnot refrigerators, in this model, the temperature of the working medium in either heat exchanging process does not need to remain constant. The model studied in this paper should be more practical and realistic than the endoreversible refrigerator one.

Combining equations (11) and (12), the optimal initial temperature of the working medium corresponding to the cold reservoir is

$$T_{c0}^{\max\chi} = \frac{1 - \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)\right]^{-1/n} + \sqrt{\frac{1}{\varepsilon_C + 1}}\left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)^{-1/n} - \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)\right]^{-1/n}\right]}{\left(1 + \sqrt{\frac{1}{\varepsilon_C + 1}}\right)\left\{1 - \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)\right]^{-1/n}\right\}} T_c \tag{15}$$

Substituting equation (15) into equation (8), we can also obtain the initial temperature T_{h0} of the working medium in the heat releasing process.

$$T_{h0}^{\max\chi} = \frac{\left\{1 + \sqrt{\frac{1}{\varepsilon_C + 1}} - \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)\right]^{-1/n} - \sqrt{\frac{1}{\varepsilon_C + 1}}\left(\frac{n\tau_c}{\Psi_c} + 1\right)^{-1/n}\right\}\left[1 - \left(\frac{n\tau_h}{\Psi_h} + 1\right)^{-1/n}\right]}{1 - \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)\right]^{-1/n} - \left(\frac{n\tau_h}{\Psi_h} + 1\right)^{-1/n} + \left[\left(\frac{n\tau_h}{\Psi_h} + 1\right)\left(\frac{n\tau_c}{\Psi_c} + 1\right)^2\right]^{-1/n}} T_h \tag{16}$$

We can obtain the temperature profiles in the heat absorbing and releasing process by substituting equation (15) into equation (2) and substituting equation (16) into equation (5).

3. Temperature profiles

We define τ/Ψ as the dimensionless contact time, which connotes the equilibrium degree of the temperature between the working medium and the heat reservoir. Larger τ/Ψ means the working medium maintains a longer contact with the heat reservoirs and will lead to a higher final temperature in the heat absorbing process and a lower temperature in the heat releasing

process. Based on equations (15) and (16), the optimal temperature profiles under different dimensionless contact time limits are analyzed as follows.

When τ_h/Ψ_h tends to infinity and τ_c/Ψ_c is finite, we obtain $T_{c0}^{\max\chi} = T_c \Big/ \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right]$. The heat releasing process is long enough that the working medium can exchange heat sufficiently with the hot reservoirs and the final temperature of the working substance is equal to that of the hot reservoir; thus, $T_{hw}(\tau_h) = T_h$ and its initial temperature is

$$T_{\mathrm{h0}}^{\max\,\chi} = \left[1 + \sqrt{\frac{1}{\varepsilon_{\mathrm{C}} + 1}} - \sqrt{\frac{1}{\varepsilon_{\mathrm{C}} + 1}}\left(\frac{n\tau_c}{\Psi_{\mathrm{c}}} + 1\right)^{-1/\mathrm{n}}\right] T_{\mathrm{h}} \tag{17}$$

which depends on the contact time of the cold reservoir. For situations where τ_c/Ψ_c tends to infinity and τ_h/Ψ_h is finite, we obtain $T_{h0}^{\max\,\chi} = \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right] T_h$. The heat absorbing process is long enough that the working medium can exchange heat sufficiently with the cold reservoirs and the final temperature of the working substance is equal to that of the cold reservoir, i.e., $T_{cw}(\tau_c) = T_c$. Moreover, its initial temperature is given by

$$T_{\mathrm{c0}}^{\max\,\chi} = \frac{\sqrt{\varepsilon_{\mathrm{C}} + 1} + \left(\dfrac{n\tau_h}{\Psi_{\mathrm{h}}} + 1\right)^{-1/\mathrm{n}}}{\sqrt{\varepsilon_{\mathrm{C}} + 1} + 1} T_c \tag{18}$$

which depends on the contact time of the hot reservoir. From equations (17) and (18), the heat releasing and absorbing processes affect one another, even though τ_h/Ψ_h or τ_c/Ψ_c tend to infinity. Furthermore, when τ_h/Ψ_h and τ_c/Ψ_c both approach infinity, we can obtain the initial temperatures, $T_{c0}^{\max\,\chi} = T_c \Big/ \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right]$ and $T_{h0}^{\max\,\chi} = \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right] T_h$ and the final temperatures $T_{cw}(\tau_c) = T_c$ and $T_{hw}(\tau_h) = T_h$ for the heat absorbing and releasing processes, respectively.

According to equations (15) and (16), when τ_h/Ψ_h approaches zero and τ_c/Ψ_c is finite, we have $T_{c0}^{\max\,\chi} = T_c$ and $T_{h0}^{\max\,\chi} = \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right] T_h$. In addition, $\tau_h/\Psi_h \to 0$ means the heat releasing processes are so short that the final temperature of the working substance is equal to its initial temperature; thus, $T_{hw}(t)^{\max\chi} = T_{h0}^{\max\chi} = \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right] T_h$. Here, $T_{c0} \leqslant T_{cw}(t) \leqslant T_c$ and $T_{h0} \geqslant T_{hw}(t) \geqslant T_h$ establish the heat absorbing and releasing processes, respectively. From these, we obtain $T_{cw}(t)^{\max\chi} = T_c$ and $T_{hw}(t)^{\max\chi} = T_h$, which mean that the temperatures of the working medium remain constant in the heat exchanging processes and the refrigerator cycle is a Carnot refrigerator cycle. However, the temperature profiles are different from those previously obtained.

When τ_c/Ψ_c approaches zero and τ_h/Ψ_h is finite, according to equations (15) and (16), we obtain $T_{c0}^{\max\chi} = T_c \Big/ \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right]$ and $T_{h0}^{\max\chi} = T_h$. Furthermore, $\tau_c/\Psi_c \to 0$ implies the heat absorbing processes are so short that the final temperature of the working substance is equal to its initial temperature; thus, $T_{cw}(t)^{\max\chi} = T_{c0}^{\max\chi} = T_c \Big/ \left[1 + \sqrt{1/(\varepsilon_C + 1)}\right]$.

Under situations where both τ_c/Ψ_c and τ_h/Ψ_h approach zero, the heat exchanging processes are so short that the final temperatures of the working medium are the same as the initial ones. Therefore, $T_{hw}(t)^{\max\chi} = T_{h0}^{\max\chi}$ and $T_{cw}(t)^{\max\chi} = T_{c0}^{\max\chi}$. Furthermore, the heat conductances are also kept constant as are the initial ones. However, the limit of equations (15) or (16), with respect to $\tau_c/\Psi_c \to 0$ and $\tau_h/\Psi_h \to 0$, does not exist if τ_c/Ψ_c and τ_h/Ψ_h are independent of each other. For further study, we consider that the dimensionless contact times fulfil a certain symmetric relation, i.e., $\tau_h/\Psi_h = \tau_c/\Psi_c$. Applying the limit $\tau/\Psi \to 0$, equations (15) and (16) yield to

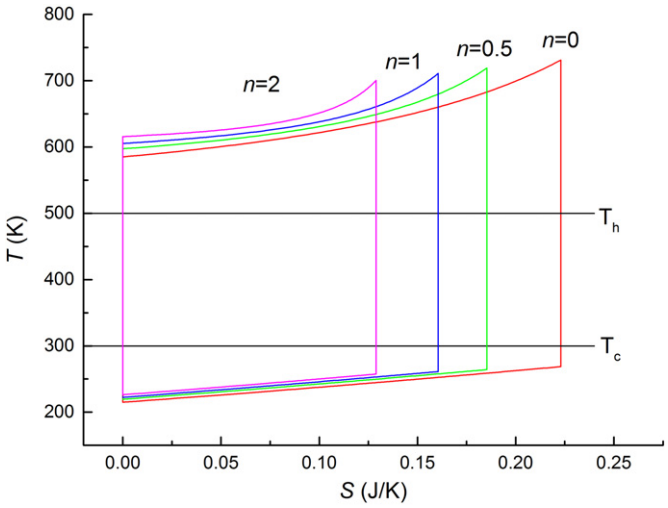

Figure 1. T–S diagrams of different optimal refrigerator cycles under different n (n = 0,0.5,1,2), where T_h = 500 K, T_c = 300 K, $\tau_h/\Psi_h = \tau_c/\Psi_c = 1$, and cm = 1J/K.

$$T_{hw}(t)^{\max\chi} = T_{h0}^{\max\chi} = \frac{2 + \sqrt{1/(\varepsilon_C + 1)}}{2} T_h \tag{19}$$

and

$$T_{cw}(t)^{\max\chi} = T_{c0}^{\max\chi} = \frac{2 + \sqrt{1/(\varepsilon_C + 1)}}{2\left[1 + \sqrt{1/(\varepsilon_C + 1)}\right]} T_c \tag{20}$$

Equations (19) and (20) have been obtained under the endoreversible Carnot refrigerator assumption with symmetric constant heat conductance [11]. Therefore, the endoreversible model is recovered under these conditions. It demonstrates again the universality of this model.

Based on the preceding analysis, the temperatures of the working medium in the heat exchanging processes will not be isothermal if τ_h/Ψ_h or τ_c/Ψ_c do not approach zero. For situations with finite dimensionless contact time, some representative cases have been selected to investigate the optimal temperature profiles during the heat exchanging processes. As depicted in figure 1, under the same dimensionless contact time, the optimal temperature profiles of the working medium move upward with increasing n. However, the optimal initial temperature of the working medium in the heat releasing process decreases with increasing n, whereas that in the heat absorbing process increases. The temperature differences between the heat reservoirs and the working medium in both heat exchanging processes are significant and cause the irreversible entropy generation. In addition, by considering various n, we could gain further insight into other refrigerator cycles, such as the reverse Brayton refrigerator, in which the temperatures of the working medium do not remain constant during the heat exchanging processes.

4. Conclusions

An analysis of the COP and its bounds at maximum χ figure of merit for refrigerators is conducted and the heat transfer processes are described using Newton's law of cooling. However, the heat conductances are no longer treated as constants, but rather as time-dependent variables i.e., $k_c = a\left(\frac{T_c - T}{T_c - T_{c0}}\right)^n$, $k_h = b\left(\frac{T - T_h}{T_{h0} - T_h}\right)^n$. The upper bound of the COP is ε_{CA} and is independent of both the time duration completing either process and the time-dependent conductance. The optimal temperature profiles in the heat exchanging processes are analyzed under different dimensionless contact time limits. When the dimensionless contact times satisfy certain relations, the endoreversible model is recovered. Furthermore, representative cases have been studied to investigate the effect of different heat conductances on temperature profiles under the maximum χ criterion. The results in the present paper could offer further insight into refrigerator cycles with isentropic compression and expansion processes.

However, the present model does not consider the internal irreversibility of the refrigerator cycles. In real life refrigerator cycles, the compressing and expanding processes are no longer isentropic and many models have been proposed to investigate the irreversibility in those processes [19, 20]. Furthermore, the impact of frictional losses on the COP are also studied for traditional and quantum refrigerators [21, 22]. Extensions of the present model concerning the internal irreversibility should be studied further.

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