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Entransy expression of the second law of thermodynamics and its application to optimization in heat transfer process

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ABSTRACT

Based on theories of thermodynamics, the energy equation in terms of entransy in heat transfer process is introduced, which not only describes the change of entransy, but also defines the entransy consumption rate. According to the regularity of entransy change in heat transfer process and the effect of entransy consumption rate on the irreversibility of heat transfer process, it can be found that entransy is a state variable, from which a new expression for the second law of thermodynamics is presented. Then by setting entransy consumption rate and power consumption rate as optimization objective and constraint condition for each other, the Lagrange conditional extremum principle is used to deduce momentum equation, constraint equation and boundary condition for optimizing flow field of convective heat transfer, which are applied to simulate convective heat transfer coupling with energy equation in an enclosed cavity. Through the numerical simulation, the optimized flow field under different constraint conditions is obtained, which shows that the principle of minimum entransy consumption is more suitable than the principle of minimum entransy consumption is more suitable than the

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1. Introduction

Past centuries have witnessed the gradual perfection of classical thermodynamics theories including irreversible thermodynamics. The second law of thermodynamics in particular, which is regarded as a fundamental law of physics, has found wide applications in engineering and scientific fields. However, in the field of heat transfer which is most closely related to thermodynamics, no important theoretical breakthrough has been achieved to heat transfer optimization over the past decades. The reason for this may be that the second law of thermodynamics has not been closely integrated with heat transfer theories in developing new concept and methodology.

In order to evaluate the degree of heat transfer enhancement, Guo afresh surveyed the mechanism for convective heat transfer and proposed the field synergy principle [1,2]. This novel concept attracted much attention of researchers [3–8]. Through their consistent effort, a systematic assessment for performance of heat transfer enhancement was gradually established. As we know that to make heat exchange equipment work efficiently with high heat transfer coefficient and lower flow resistance or lower energy consumption, the key factor is to optimize heat transfer process. For this purpose, Guo et al. newly proposed a new physical quantity "entransy" based on the analogy between heat conduction and electrical conduction as well as thermodynamics theories [9]. They deducted this concept theoretically and validated it through modeling and numerical computation [10–16]. Although introduction of the concept of entransy has shown the advantages in heat transfer optimization, it is still in its initial stage and needs to be perfected. By resorting to the second law of thermodynamics, this paper attempts to give new insights into the concept of entransy which is treated as a basic physical quantity, and to reaffirm the importance of entransy in developing optimization theories for heat transfer process.

2. The Irreversibility of heat transfer process

The transfer of energy, momentum and mass are three main forms of transport phenomena in the nature or engineering. Energy takes many forms, such as thermal energy, mechanical energy, electronic energy, optical energy, sound energy, etc. When energy in any form other than thermal energy is transferred, a part of it will be transformed into thermal energy, and when thermal energy is transferred, a certain amount of it will lose or dissipate to somewhere. This is the so-called irreversibility of energy conversion and transport processes. Therefore, in order to reduce the irreversibility of heat transfer process, it is necessary to explore its physical mechanism and optimize the process by quantitatively analyzing the changes of physical quantities and recording the traces left by these changes.

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Nomenclature

A, B, C ₀	Lagrange multipliers	t	time [s]	
С	specific heat [J/(kg · K)]	U	velocity vector [m/s]	
Cp	specific heat at constant pressure [J/(kg · K)]	V	volume [m ³]	
ĥ	enthalpy [J/kg]	Ζ	entransy [J · K]	
J, J′	functional	Z	specific entransy [J · K/kg]	
p	pressure [Pa]	$Z_{e,T}$	entransy consumption [J · K]	
Q	heat flux [W]			
<i>Q'''</i>	internal heat source [W/m ³]	Greek s	Greek symbols	
q	heat flux vector [W/m ²]	λ	thermal conductivity [W/(m · K)]	
S	entropy [J/K]	ρ	fluid density [kg/m ³]	
S	specific entropy [J/(kg · K)]	μ	viscosity coefficient [kg/(m · s)]	
$S_{g,p}$	entropy generation induced by fluid viscosity [J/K]	Φ	dissipated heat from fluid viscosity [W/m ³]	
$S_{g,T}$	entropy generation induced by heat transfer [J/K]	Ω	control volume [m ³]	
Т	temperature [K]			

2.1. Thermodynamic definition of entransy and its deduction

For a non-equilibrium convective heat transfer process, its energy equation can be expressed in terms of enthalpy as:

$$\rho \frac{Dh}{Dt} = -\nabla \cdot \boldsymbol{q} + \boldsymbol{\Phi} + \dot{\mathbf{Q}}^{\prime\prime\prime}. \tag{1}$$

Where ρ is the fluid density, *h* is the fluid enthalpy, $-\nabla \cdot \boldsymbol{q}$ is the heat flux transferring into and out of fluid elementary volume, $\boldsymbol{\Phi}$ is the dissipated heat from fluid viscosity, and $\dot{Q}^{\prime\prime\prime}$ is the internal heat source.

Eq. (1) can be rewritten in terms of entropy as:

$$\rho \frac{Ds}{Dt} = -\nabla \cdot \left(\frac{\mathbf{q}}{T}\right) + \frac{\lambda (\nabla T)^2}{T^2} + \frac{\Phi}{T} + \frac{\dot{Q}^{\prime\prime\prime\prime}}{T}.$$
(2)

Where *s* is the entropy of fluid, $-\nabla \cdot \left(\frac{q}{T}\right)$ is the entropy flow transferring into and out of fluid elementary volume, $\frac{\lambda(\nabla T)^2}{T^2}$ is the entropy generation rate induced by heat transfer process, $\frac{q}{T}$ is the entropy generation rate caused by the heat generation from viscous dissipation of mechanical energy, which can also be defined as analogical entropy source, and $\frac{\dot{Q}'''}{T}$ is the internal heat source in form of entropy expression. The differential entropy for incompressible fluid can be expressed as: $ds = \frac{dh}{T} = \frac{cdT}{T}$.

The principle of entropy generation minimization is widely utilized to optimize the thermodynamics process based on the second law of thermodynamics [17]. According to finite time thermodynamics theory [18], the entropy generation rate induced by heat transfer is a representation of the irreversibility of the process, so it can be denoted as the dot product of entropy flow and force, i.e. $\frac{\lambda(\nabla T)^2}{T^2} = \frac{q}{T} \cdot \left(-\frac{\nabla T}{T}\right)$, while the entropy generation rate $\frac{\phi}{T}$ denotes mechanical energy dissipation induced by fluid viscosity. Accordingly, thermodynamic equilibrium of entropy is given as:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{g,T} + S_{g,p}.$$
 (3)

Where the integral term represents entropy flow, $S_{g,T}$ represents entropy generation induced by heat transfer, and $S_{g,p}$ represents entropy generation induced by fluid viscosity. When boundary heat flux is zero, entropy flow is zero. Then Eq. (3) reduces to:

$$\Delta S = S_{g,T} + S_{g,p}.\tag{4}$$

Multiplying both sides of Eq. (2) by T^2 and making transformation yields:

$$\rho cT \frac{DT}{Dt} = -\nabla \cdot (\boldsymbol{q}T) - \lambda (\nabla T)^2 + \Phi T + \dot{Q}^{\prime\prime\prime} T.$$
(5)

Referencing to the definition of entropy, the entransy of incom-

Referencing to the definition of entropy, the entransy of incompressible fluid, termed *z*, can be defined in the following differential expression:

$$dz = Tdh = cTdT. (6)$$

It can be seen that Eq. (6) is a differential definition for entransy similar to thermodynamics entropy, which shows that the relation between differential enthalpy and temperature is expressed as product other than quotient. Thus the energy equation of convective heat transfer can be expressed in terms of entransy as:

$$\rho \frac{Dz}{Dt} = -\nabla \cdot (\boldsymbol{q}T) - \lambda (\nabla T)^2 + \boldsymbol{\Phi}T + \dot{\boldsymbol{Q}}'''T.$$
⁽⁷⁾

Where z is the entransy of fluid, $-\nabla \cdot (\mathbf{q}T)$ is the entransy flow transferring into and out of fluid elementary volume, $\lambda(\nabla T)^2$ is the entransy consumption rate induced by heat transfer process, ΦT is defined as the analogical entransy source induced by dissipated heat from fluid mechanical energy, and $\dot{Q}''T$ is the internal heat source in form of entransy expression.

As we know that a state variable or parameter can be defined to represent the state of a thermodynamics system. If the values of all state variables of a system are known, the state of this system can be well described. The entransy is such a variable introduced to express the nature of a system, in which the heat is transferred from the high to the low temperature sites. The entransy consumption rate is induced by heat transfer temperature difference, which can



Fig. 1. Sketch of an enclosed cavity with geometry and boundary conditions (L = H = 15 mm).



Fig. 2. Vortex structure and optimized fields with minimum entransy consumption ($C_0 = -2.0 \times 10^{-8}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

be denoted as the dot product of entransy flow and force, i.e. $\lambda(\nabla T)^2 = \mathbf{q}T \cdot (-\frac{\nabla T}{2})$. Clearly, $\lambda(\nabla T)^2$ and ΦT are totally different, the former reflects the irreversibility of thermal energy transport process, while the latter reflects the irreversibility of mechanical energy dissipation. Since the value of ΦT is much less than that of $\lambda(\nabla T)^2$, it can be neglected in the case of simplification.

Corresponding to Eq. (7), thermodynamic equilibrium of entransy can be expressed as:

$$\Delta Z = Z_2 - Z_1 = -\int_1^2 T \delta Q - Z_{e,T}.$$
 (8)

Where the integral term is entransy flow, $Z_{e,T}$ is entransy consumption. When boundary heat flow is zero, entransy flow is zero. Then Eq. (8) reduces to:

$$\Delta Z = -Z_{e,T}.\tag{9}$$

If the motion is excluded from consideration, the amount of viscous dissipation is zero, and heat transfer process can be regarded as pure heat conduction. Then enthalpy equation is:

$$\rho \frac{\partial h}{\partial t} = -\nabla \cdot \boldsymbol{q} + \dot{Q}^{\prime\prime\prime},\tag{10}$$

entropy equation is:

$$\rho \frac{\partial s}{\partial t} = -\nabla \cdot \left(\frac{\mathbf{q}}{T}\right) + \frac{\lambda (\nabla T)^2}{T^2} + \frac{\dot{Q}'''}{T},\tag{11}$$

and entransy equation is:

$$\rho \frac{\partial z}{\partial t} = -\nabla \cdot (\boldsymbol{q}T) - \lambda (\nabla T)^2 + \dot{Q}'''T.$$
(12)

Therefore we can say that, for any heat transfer process, changes in enthalpy take place when heat flow is transported; changes in entropy take place when entropy flow is transported, and the changes is tracked by the entropy generation rate; changes in entransy take place when entransy flow is transported, and the changes is tracked by entransy consumption rate. Thus, the goal of reducing transport process irreversibility and enlarging heat transfer efficiency actually becomes to optimize entransy consumption rate $\lambda(\nabla T)^2$.



Fig. 3. Vortex structure and optimized fields with minimum entransy consumption ($C_0 = -1.0 \times 10^{-7}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

2.2. A new statement for the second law of thermodynamics

In non-equilibrium heat transfer process, both entropy and entransy would change as variables. From Eqs. (2) and (7), we know that entropy in an irreversible process tends to increase while entransy changes in the opposite direction. The trace left in a transport process over time and space takes the form of entropy generation rate or entransy consumption rate. The higher the degree of irreversibility is, the larger the entropy generation rate or entransy consumption rate would be. Therefore, entransy, similar to entropy, is actually a state variable or parameter reflecting the irreversibility of a transport process, from which a new statement for the second law of thermodynamics would be expressed as: entransy never increases when heat is transferred from higher to lower temperature in non-equilibrium or equilibrium state. This statement can be called as the principle of entransy decrease in heat transfer process. For a system in non-equilibrium state, entransy tends to decrease over time until the system reaches at equilibrium state; and for a system in equilibrium state, when heat

is transferred from heat source at high temperature to heat sink at low temperature, entransy will decrease as well.

The principle of entransy decrease corresponds to the principle of entropy increase. It is well known that entropy is a measure of the degree of disorder of a system in the micro-level. The larger the entropy is, the more chaotic the microscopic motion of molecules or particles will be. By analogy with this microscopic explanation for entropy, entransy can be regarded as a physical quantity measuring the degree of order of a system in the micro-level. When the entransy of a system is small, it means that the system is not well ordered and its capability to transfer thermal energy is not high. Conversely, an optimized system will be in good order and will have higher heat transfer capability. We take a gas system with high temperature as an example. When the system transfers heat to its surroundings, the gas temperature in the system drops, and the molecular motion in the gas becomes more and more disordered, so that the system becomes less and less ordered. When the gas temperature is lowered to the temperature of surroundings, the degree of order of the system tends to be least,



Fig. 4. Vortex structure and optimized fields with minimum entransy consumption ($C_0 = -5.0 \times 10^{-7}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

and its heat transfer capability approximates zero. The entransy of the system will be least at this time.

3. Application of the principle of entransy decrease to heat transfer optimization

Optimizing a heat transfer process is to reduce the irreversibility of the process by controlling the entransy consumption and the degree of order of the system. In order to optimize flow fields in different channels, the Lagrange conditional extremum principle is used to construct functional. Then by functional variation and seeking extremum, the momentum equation, constraint equation and boundary conditions for the optimized flow field are obtained.

3.1. Optimization method

The principle of minimum entropy generation proposed for optimizing the heat-work conversion process contributes to the optimization of the thermal energy transport process. From Eq. (2), one can find that the entropy generation rate is positively correlated with temperature gradient and negatively correlated with absolute temperature. However, Eq. (7) shows that entransy consumption rate is only correlated with temperature gradient. Lower entransy consumption rate ensures smaller temperature gradient, which will even up temperature profile of the fluid, reduce heat resistance, and thereby enhance convective heat transfer. For the problem of heat transfer optimization, we wish the temperature field of the fluid to be uniform. This can be achieved by minimizing temperature gradient ∇T or entransy consumption rate $\lambda(\nabla T)^2$, but not by minimizing entropy generation rate and entransy consumption rate reflects the process irreversibility, the principle of minimum entransy consumption is more appropriate for the optimization of the heat transfer process.

Enhancing convective heat transfer and reducing fluid flow resistance are two aspects of improving the performance of heat transfer equipment, but they always contradict each other. Enhancing convective heat transfer may lead to the increase in flow resistance, and much higher flow resistance may weaken convective heat transfer, which implies that there exists some kind



Fig. 5. Vortex structure and optimized fields with minimum entransy consumption ($C_0 = -1.0 \times 10^{-6}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

of synergetic relation between them. Therefore, when constructing functional according to the Lagrange conditional extremum principle, two aspects need to be considered: (1) to set minimum entransy consumption rate as optimization objective and power consumption rate as constraint condition; (2) to set minimum power consumption rate as optimization objective and entransy consumption rate as constraint condition.

3.2. Optimization equation for convective heat transfer

For convective heat transfer process, the fluid momentum equation is:

$$\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\nabla \boldsymbol{p} + \mu \nabla^2 \boldsymbol{U}. \tag{13}$$

Multiplying both sides of Eq. (13) by velocity vector **U** yields:

$$-\nabla p \cdot \boldsymbol{U} = (\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} - \mu \nabla^2 \boldsymbol{U}) \cdot \boldsymbol{U}. \tag{14}$$

Where $-\nabla p \cdot U$ represents the work consumed by the fluid. Eq. (14) reflects the conservation of mechanical energy, i.e. the pump power consumed by the fluid is the sum of kinetic energy loss and viscous

dissipation power. Thus, if power consumption is small, the kinetic energy loss and viscous dissipation power are both small in the system. In the mean time, if the viscous dissipation can be reduced, the irreversibility of the process will be reduced.

For a heat transfer process in any channel, if the optimization objective is to achieve the temperature uniformity of fluid, then with minimum entransy consumption rate and fixed power consumption rate, the Lagrange function can be made by variational calculus:

$$J = \int_{\Omega} \left[\lambda (\nabla T)^{2} + C_{0} (\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} - \mu \nabla^{2} \boldsymbol{U}) \cdot \boldsymbol{U} + A \nabla \cdot \boldsymbol{U} \right. \\ \left. + B (\lambda \nabla^{2} T - \rho \boldsymbol{c}_{p} \boldsymbol{U} \cdot \nabla T) \right] dV.$$
(15)

Where A, B and C_0 are Lagrange multipliers. Multiplier C_0 , thermal conductivity λ , fluid density ρ , specific heat c_p and viscosity coefficient μ are taken as constant respectively.

By finding functional variation with respect to velocity U and temperature T respectively, the momentum equation is obtained as:



Fig. 6. Vortex structure and optimized fields with minimum power consumption ($C_0 = -3.0 \times 10^{-5}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

$$\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\nabla p + \mu \nabla^2 \boldsymbol{U} + \frac{\rho c_p B}{C_0} \nabla T, \qquad (16)$$

where $\frac{\rho c_p B}{C_0} \nabla T$ denotes the virtual volume force induced to optimize flow field, and the constraint equation of parameter *B* is deduced as:

$$\rho c_p \boldsymbol{U} \cdot \nabla \boldsymbol{B} + \lambda \nabla^2 \boldsymbol{B} - 2\lambda \nabla^2 \boldsymbol{T} = \boldsymbol{0}. \tag{17}$$

For Eq. (17), the boundary condition at constant wall temperature is:

$$B=0,$$
 (18)

and the boundary condition at constant heat flux is:

$$2\lambda \nabla T - \lambda \nabla B = 0. \tag{19}$$

In addition, if the optimization objective is to reduce flow resistance of the fluid, we can construct Lagrange function with minimum power consumption rate and fixed entransy consumption rate as:

$$J' = \int_{\Omega} \left[(\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} - \mu \nabla^2 \boldsymbol{U}) \cdot \boldsymbol{U} + C_0 \lambda (\nabla T)^2 + A \nabla \cdot \boldsymbol{U} \right. \\ \left. + B(\lambda \nabla^2 T - \rho c_p \boldsymbol{U} \cdot \nabla T) \right] dV.$$
(20)

After finding functional variation with respect to velocity U and temperature T respectively, we can have the momentum equation as:

$$\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\nabla p + \mu \nabla^2 \boldsymbol{U} + \rho \boldsymbol{c}_p \boldsymbol{B} \nabla \boldsymbol{T}, \qquad (21)$$

where $\rho c_p B \nabla T$ represents the virtual volume force, similar to the one in Eq. (16), and the constraint equation of parameter *B* as:

$$\rho c_p \boldsymbol{U} \cdot \nabla \boldsymbol{B} + \lambda \nabla^2 \boldsymbol{B} - 2C_0 \lambda \nabla^2 \boldsymbol{T} = \boldsymbol{0}.$$
⁽²²⁾

For Eq. (22), the boundary condition at constant wall temperature is:

$$B = 0, \tag{23}$$

and the boundary condition at constant heat flux is:

$$2C_0\lambda\nabla T - \lambda\nabla B = 0. \tag{24}$$



Fig. 7. Vortex structure and optimized fields with minimum power consumption ($C_0 = -4.0 \times 10^{-4}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

The coupling of above momentum equation, constraint equation and energy equation makes it possible to simulate convective heat transfer in any channel with different boundary conditions. Based on the simulation results, the optimal flow field structure depending on different optimization objectives and constraint conditions can be obtained to achieve the ultimate goal of enhancing convective heat transfer and controlling fluid flow resistance, which may guide the design for heat transfer unit and heat exchanger.

4. Calculation and analysis for the optimized flow field in an enclosed cavity

A 2D enclosed cavity model [19] with geometric size of $15 \times 15 \text{ mm}^2$ is shown in Fig. 1. The left and right wall temperatures are constant: $T_1 = 300K$, $T_h = 315K$. The upper and lower walls are in adiabatic condition: $q_w = 0$. Normally speaking, in an enclosed cavity with constant wall temperature or constant heat flux, the driving force for convective heat transfer comes from the density difference of the fluid. Buoyancy force, however, is

not included in the model we established, the fluid is driven by a virtual volume force induced by heat transfer optimization. Thus, by solving above controlling equations, we can inspect the effect of virtual force field on the flow organization and optimization to validate the model. Noting that the value of coefficient C₀ is related to the intensity of virtual force field, we find that the magnitude of C₀ value would result in different flow filed structure. In addition, since C₀ value also reflects the degree of constraint, it is restricted to a certain range depending on the geometric size and thermal boundary conditions. The FLUENT 6.3 is used to the simulation. The velocity and pressure are linked using the SIMPLEC algorithm. The convection and diffusion terms are discretized using the QUICK scheme. The user defined function (UDF) in the FLUENT is utilized for solving the governing Eqs. (16) and (21) respectively. The convergence solutions are obtained, when the residuals of all the coupled equations are less than 10^{-8} .

Figs. 2–5 show the calculation results for the case in which minimum entransy consumption is set as optimization objective and fixed power consumption as constraint condition. The cavity sizes are chosen as non-dimension in the form of center symmetry. In the calculation, a different value for C_0 is taken. It can be found that



Fig. 8. Vortex structure and optimized fields with minimum power consumption ($C_0 = -6.0 \times 10^{-4}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.

with the increase in absolute value of C_0 , the number of vortex will increase from one to four, and if absolute value of C_0 keeps growing, more vortexes would appear as a result of growing virtual volume force. Apparently, the increase in vortex number would intensify the fluid disturbance, which would even up temperature profile, leading to enhancing heat transfer. Accordingly, the virtual volume force is acting as a vortex generator in the enclosed cavity without considering buoyancy force.

Figs. 6–9 show the calculation results for the case in which minimum power consumption is set as optimization objective and fixed entransy consumption as constraint condition. It can be found that, similar to the case in Figs. 2–5, with the increase in absolute value of C_0 , the vortex number inside the enclosed cavity will increase from one to four, and more vortexes would appear when C_0 keeps growing. By comparing the two cases, we can find that whether the optimization objective is minimum entransy consumption or minimum power consumption, the optimization results do not differ greatly. If the comparison is made closely, it can be found that temperature distribution in the case of minimum power consumption appears more even and heat transfer in this case is more effective.

Fig. 10(a) shows the change curve of heat transfer capacity in the enclosed cavity when different optimization objectives are set. If there is no volume force acting inside, the process is pure heat conduction with heat transfer capacity of 0.365W. If the optimization objective is minimum entransy consumption, compared to the process of pure heat conduction, heat transfer capacity increases from 97% to 611%. If the optimization objective is minimum power consumption, heat transfer capacity increases from 111% to 683%. Fig. 10 (b) shows the change curve of Nu number with different optimization objective. In case of pure heat conduction, equivalent Nu number is 1. When certain optimization objective is taken, Nu number will be larger than 1, which indicates a certain degree of heat transfer enhancement. After comparing between Figs. 10(a) and 10(b), we find that under the same vertex number in the enclosed cavity, heat transfer capacity with minimum power consumption as optimization objective is greater than that with minimum entransy consumption as optimization



Fig. 9. Vortex structure and optimized fields with minimum power consumption ($C_0 = -1.0 \times 10^{-3}$). (a) Stream function field; (b) temperature field; (c) constraint variable field; (d) pressure field.



Fig. 10. Heat flux (a) and Nu number (b) under different optimization objective. C_{0,1}: for minimum power consumption rate; C_{0,2}: for minimum entransy consumption rate.

objective. This suggests that to effectively reduce flow resistance is conducive to enhancing heat transfer.

5. Conclusions

Entransy is a state variable that measures the degree of order of a system in which heat is transferred. Entransy consumption rate reflects the degree of irreversibility of heat transfer process, which can be expressed as a dot product of entransy flow and force. The principle of entransy decrease in heat transfer process states that entransy would never increase when heat is transferred from higher to lower temperature in the non-equilibrium or equilibrium state.

Since entransy consumption rate is only correlated with temperature gradient of the fluid, and temperature uniformity is one of most important objective pursued by convective heat transfer optimization, the principle of minimum entransy consumption is accordingly more suitable for optimizing heat transfer process than the principle of minimum entropy generation. The lower the entransy consumption rate is, the better the temperature uniformity of the flow field will be, and the less the irreversibility of convective heat transfer process will be.

As power consumption of the fluid is both related with flow resistance and momentum loss, to achieve minimum power consumption is considered as an appropriate optimization objective. The lower the power consumption rate is, the less the flow resistance will be, and the smaller the momentum loss will be, which leads to enhancing heat transfer.

When optimizing a heat transfer process, entransy consumption rate and power consumption rate may not be reduced simultaneously. By setting minimum power consumption rate as optimization objective and entransy consumption rate as constraint condition, better optimization results can be obtained than the case by setting minimum entransy consumption rate as optimization objective and power consumption rate as constraint condition, for the parameters selected in the paper, such as Re number, geometric sizes and boundary conditions, etc.

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