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Mechanism and numerical analysis of heat transfer enhancement in the core flow along a tube

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The present study introduces the principles of enhanced heat transfer in the core flow to form an equivalent thermal boundary layer in the fully developed laminar tube flow, which consequently enlarges the temperature gradient of the fluid near the tube wall, and thereby enhances the heat transfer between the fluid and the tube wall. At the same time, the increase of flow resistance in the tube is not so obvious. Mechanism analysis and numerical calculation based on air and water have been carried out to verify the principle and method presented in this paper, which may bring positive effects to the design of heat exchanger with high heat transfer efficiency and low flow resistance.

laminar tube flow, core flow, boundary flow, heat transfer enhancement

1 Introduction

As we know that an effective way of intensifying heat convection of laminar or turbulent flow across a flat plate is simply to raise fluid velocity, so that hydrodynamic and thermal boundary layers become thinner, and temperature gradient of fluid near the wall becomes larger. When laminar fluid flows in a tube, however, there is no boundary layer in the fully developed flow except in the entrance length of tube, as temperature gradient and velocity profile vary obviously in the cross section of tube. In fact, for the fully developed laminar tube flow, *Nu* number is 4.364 for constant heat flux and 3.657 for constant wall temperature with no change[1]. Therefore, it is not suitable to follow this way of improving the performance of a heat exchanger with tubes or channels just by increasing fluid velocity.

In a tube, flow field can be divided into two parts, i.e. core flow and boundary flow[2]. The common methods about heat transfer enhancement in a tube are as follows[3]: 1) disrupting the fluid

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boundary layer near the wall; 2) extending the solid surface to transfer more heat flux; 3) changing the physical nature of the surface, etc. Since these kinds of enhanced heat transfer are focused on the boundary they are classified as the boundary (or surface) enhancement[4]. However, although these measures are quite effective to transfer more heat, a fact that viscous resistance of fluid is initiated from the wall surface could not be avoided, that is to say the cost of increase in flow resistance must be paid to achieve a better effect of heat convection.

After taking some measures of heat transfer enhancement such as extended ribs, vortex generator and groove in the inner surface of tube, the flow resistance inside tube will increase remarkably because the increase in fluid velocity gradient, viscous diffusion and momentum dissipation near the boundary makes the shear force, the friction between fluid and surface and the fluid dissipation work increase more or less. According to this analysis, ref. [5] and footnote 1) indicated that heat transfer could be enhanced in the core flow by forming an equivalent thermal boundary layer in the fully developed tube flow. Based on this concept, we further propose a new method—heat transfer enhancement in the core flow, which means reducing flow resistance near the boundary as much as possible and taking various effective measures of heat transfer enhancement in the core flow zone to achieve heat transfer enhancement inside a tube.

2 Theory and model for heat transfer enhancement in the core flow

To be different from traditional boundary flow enhancement, we think that the core flow inside a tube is an important zone for heat transfer enhancement which is worth to fully utilize, and the core flow and the boundary flow can be coupled to achieve the compound heat transfer enhancement. To produce remarkable heat transfer effect, the most direct method is to make the core flow temperature inside a tube uniform as much as possible so as to form an equivalent thermal boundary layer which has a larger temperature gradient near the surface of tube. At the same time, some further considerations are as follows: reducing velocity gradient inside tube as much as possible to avoid too large fluid shear force; reducing the disturbance to the hydrodynamic boundary inside tube as much as possible to avoid too large fluid momentum loss; breaking the continuously extended surface as much as possible to avoid too large surface friction. Therefore, the essential of this method can be mainly summarized as: 1) making temperature uniform in the core flow; 2) not obviously raising velocity gradient in the flow field; 3) not disrupting fluid near the boundary; 4) not extending continuous surface on the wall.

2.1 Theoretical modeling

Mohamad[6] reported heat transfer enhancement in the heat-exchanger tubes partially filled with porous media, and studied the influence of the porous radius ratio for the flow and heat transfer characters in the tube. Other researchers studied the heat transfer enhancement method by filling a pipe completely with metal foam, and numerically calculated the heat flux conducted by the metal foam with two-equation non-equilibrium model. Their conclusions showed that metal foam filled in a pipe enhanced heat transfer greatly at the cost of a big pressure drop.

An instance is schematically presented in Figure1, in which the fluid with uniform inlet velocity

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1) Liu W, Yang K, Nakayama A. Enhancing heat Transfer in the core flow by forming an equivalent thermal boundary layer in the fully developed tube flow. Sixth International Conference on Enhanced, Compact and Ultra-Compact Heat Exchangers: Science, Engineering and Technology, Potsdam, Germany, 2007

and temperature flows through a tube partially filled with porous media, and is heated by the tube wall at constant and uniform heat flux.

In order to obtain the mathematic model, the following assumptions are made: (1) homogeneous and isotropic porous material; (2) medium with no distension or contraction; (3) subject to local thermal equilibrium throughout analysis domain; (4) steady-state laminar flow is considered.

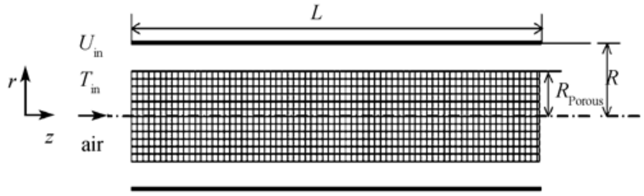

Figure 1 Schematic structure of calculation model.

The mathematic model and its boundary conditions are described as follows[7]:

Continuity equation:

$$\frac{\partial}{\partial z}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho v)=0. \tag{1}$$

Momentum equations in porous region:

$$\begin{aligned}\frac{1}{\varepsilon^2}\frac{\partial}{\partial z}(\rho uu)+\frac{1}{r\varepsilon^2}\frac{\partial}{\partial r}(r\rho vu)=&-\frac{\partial p}{\partial z}-\frac{\mu u}{K}-\frac{\rho F}{\sqrt{K}}\sqrt{u^2+v^2}\ u\\&+\frac{1}{\varepsilon}\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\frac{1}{r\varepsilon}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right),\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{1}{\varepsilon^2}\frac{\partial}{\partial z}(\rho uv)+\frac{1}{r\varepsilon^2}\frac{\partial}{\partial r}(r\rho vv)=&-\frac{\partial p}{\partial r}-\frac{\mu v}{K}-\frac{\rho F}{\sqrt{K}}\sqrt{u^2+v^2}\ v\\&+\frac{1}{\varepsilon}\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)+\frac{1}{r\varepsilon}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\frac{\mu}{\varepsilon}\frac{v}{r^2}.\end{aligned} \tag{3}$$

Momentum equations in non-porous region:

$$\frac{\partial}{\partial z}(\rho uu)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho vu)=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right), \tag{4}$$

$$\frac{\partial}{\partial z}(\rho uv)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho vv)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\mu\frac{v}{r^2}. \tag{5}$$

Energy equation in porous region:

$$\frac{\partial}{\partial z}(\rho cuT)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho cvT)=\frac{\partial}{\partial z}\left(k_{\text{eff}}\frac{\partial T}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rk_{\text{eff}}\frac{\partial T}{\partial r}\right). \tag{6}$$

Energy equation in non-porous region:

$$\frac{\partial}{\partial z}(\rho cuT)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho cvT)=\frac{\partial}{\partial z}\left(k_f\frac{\partial T}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rk_f\frac{\partial T}{\partial r}\right). \tag{7}$$

Where K is permeability defined as $K=d_p^2\varepsilon^3/[150(1-\varepsilon)^2]$, ε is porosity, and k_{eff} is effective thermal conductivity defined as $k_{\text{eff}}=(1-\varepsilon)k_s+\varepsilon\, k_f$.

Boundary conditions:

$$z=0\text{:}\ \ u=u_{\text{in}},\ \ v=0,\ \ T=T_{\text{in}};$$

$$z=\text{L}\text{:}\ \ \frac{\partial u}{\partial z}=0,\ \ v=0,\ \ \frac{\partial T}{\partial z}=0;$$

$$r = 0:\ \frac{\partial u}{\partial r} = 0,\ v = 0,\ \frac{\partial T}{\partial r} = 0;$$

$$r = \mathrm{R}:\ u = 0,\ v = 0,\ k_f \frac{\partial T}{\partial r} = q.$$

Equations (1)—(7) with boundary conditions were solved by SIMPLE algorithm to demonstrate velocity, temperature and pressure fields. After finding out velocity and temperature fields, heat transfer coefficient of tube or channel flow can be calculated as

$$h = \frac{q}{T_w - T_m},$$

where T_m is fluid bulk temperature inside tube or channel:

$$T_m = \frac{\int_0^R uTr\mathrm{d}r}{\int_0^R ur\mathrm{d}r}.$$

Nusselt number and friction factor for a tube or channel can be calculated as

$$Nu = \frac{2hR}{k_f}, \quad f = \frac{4\frac{\mathrm{d}p}{\mathrm{d}z}R}{\rho u_m^2},$$

where u_m is cross-sectional average velocity defined as

$$u_m = \frac{\int_0^R 2\pi ur\mathrm{d}r}{\pi R^2}.$$

To value the effect of heat transfer enhancement under given pumping power, the formula of performance evaluation criteria is employed as

$$PEC = \frac{Nu / Nu_{\text{free}}}{(f / f_{\text{free}})^{1/3}}, \tag{8}$$

where Nu_{free} and f_{free} are Nusselt number and friction factor for a tube without porous media respectively.

2.2 Results and discussion

The calculated results based on the above model are shown in Figures 2 —11. The calculation parameters have been chosen for Reynolds number 500, thermal conductivity of porous material 200 W/(m·℃). The fluids in the calculation are air and water.

The dimensionless velocity and temperature profiles are plotted in Figures 2 and 3. Where the fluid is air, the porosity is 0.98, R_{rad} is the porous radius ratio ($R_{\text{rad}} = R_{\text{Porous}}/R$; R_{Porous} is the radius occupied by the porous media), the dimensionless temperature is defined as $\theta = (T_w - T)/(T_w - T_m)$. As shown in the figures, when the R_{rad} is large (R_{rad}=0.8–0.96), the increase in the velocity gradient is not obvious, the temperature profile is very even in the core flow, and an equivalent thermal boundary layer with large temperature gradient appears near the tube wall.

Figures 4 and 5 display the variation of Nu number and friction factor at different R_{rad} for air. As shown in the figures, the increase in the porosity is helpful to decrease in the flow resistance, but leads to decrease in heat transfer ability. However, the influence of the porosity on heat transfer is less than that on flow resistance. On the other hand, increase in the porous radius ratio will enhance heat transfer, but also leads to increase in flow resistance. In momentum equations (2) and (3), Darcy resistance represents the friction from porous mixture to fluid, while Forchheimer inertial drag is direct proportion to the square of fluid velocity. When the porosity increases, the specific surface area of porous material will decrease, thus the friction will also decrease. However, as the fluid velocity increases, the inertial drag will increase. The total effect is that, with the increase in porosity, the decrease in Darcy resistanc is more obvious than the increase in inertial drag. In addition, the increase in both porosity and porous radius ratio will lead to the decrease in flow resistance to a certian extent.

The dimensionless velocity and temperature profiles are plotted in Figures 6 and 7. Where the fluid is water, the porosity is 0.98. Compared with the results calculated from air, the level of temperature uniformity for water decreases, thus the temperature gradient near the tube wall decreases. This phenomenon is related to the difference between the effective thermal conductivity k_{eff} and fluid thermal conductivity k_f. The increase in this difference is helpful to enhancing heat transfer. This phenomenon also shows that heat transfer enhancement in the core flow is good for fluid with low *Pr* number.

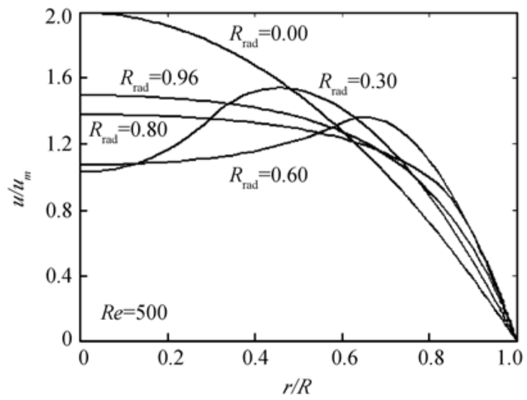

Figure 2 Velocity profiles in the fully developed flow (air).

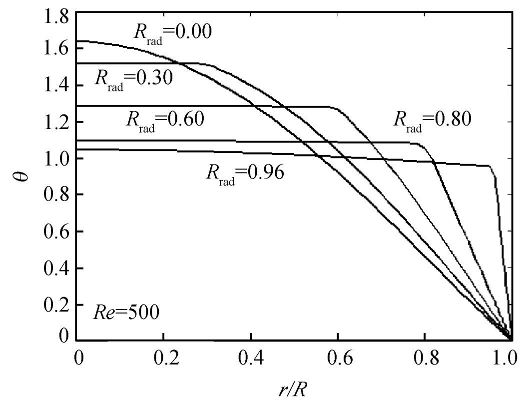

Figure 3 Temperature profiles in the fully developed flow (air).

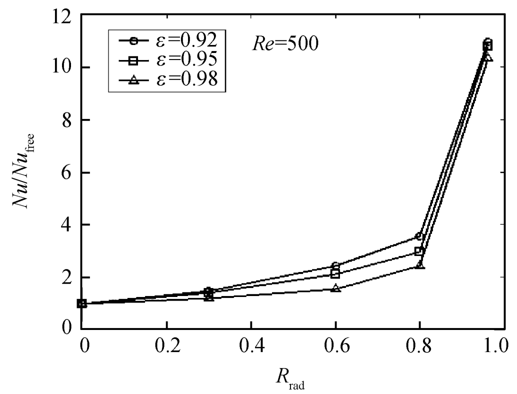

Figure 4 *Nu* number in the fully developed flow (air).

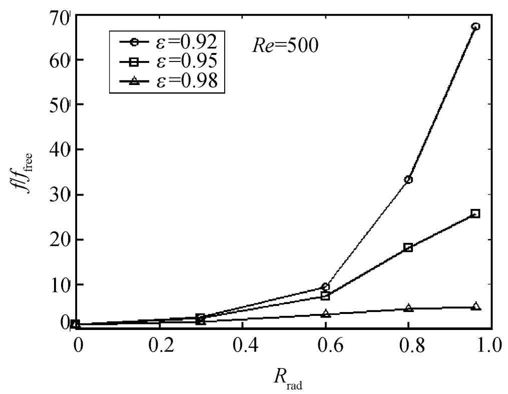

Figure 5 Friction factor in the fully developed flow (air).

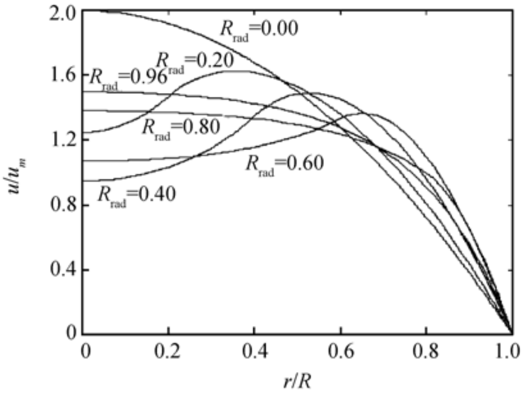

Figure 6 Velocity profiles in the fully developed flow (water).

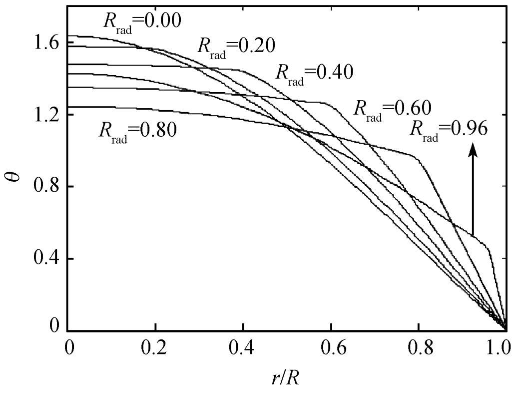

Figure 7 Temperature profiles in the fully developed flow (water).

Figures 8 and 9 display the variation of *Nu* number and friction factor at different R_{rad} for water. As shown in the figures, increase in the porous radius ratio will enhance heat transfer and raise flow resistance obviously. However, when both porous radius ratio and porosity are high, the heat transfer can be enhanced, and the flow resistance can be decreased at the same time. These results are consistent with that shown in Figures 4 and 5.

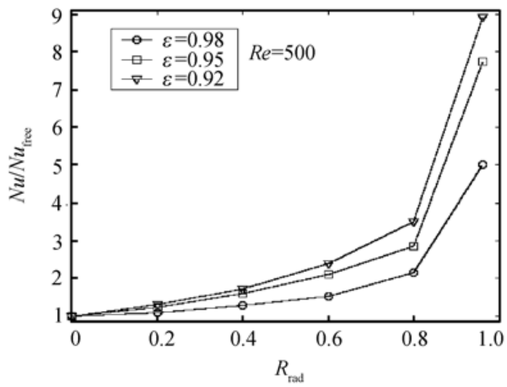

Figure 8 *Nu* number in the fully developed flow (water).

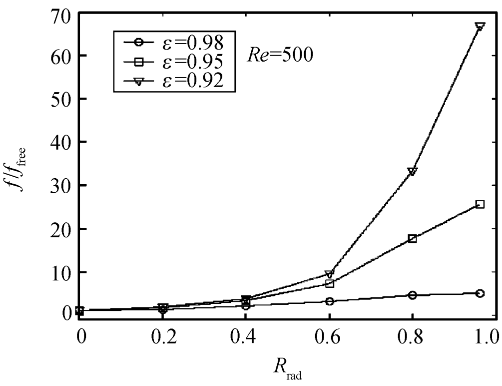

Figure 9 Friction factor in the fully developed flow (water).

Figures 10 and 11 display the variation of *PEC* value at different R_{rad} for air and water respectively. As shown in the figures, with the increase in both porous radius ratio and porosity, the rate of the increase in heat transfer to the increase in friction factor will rise.

Although the drawing lines in Figures 2—11 are obtained from the fully developed section of a tube, the principle of heat transfer enhancement in the core flow proposed in the present paper is also applicable to the developing section of a tube, because the method that porous media is filled in the developing section of a tube will also make the core flow temperature in the most zone uniform, thereby to transfer more heat. In addition, from the view of engineering application, although the sintered or foamed type metal porous media with high porosity and high porous radius ratio is a little bit difficult to manufacture and will cost more, it is possible to utilize a kind of porous material with ε=0.92, R_{rad}=0.95, and relatively low cost in the practical engineering. For

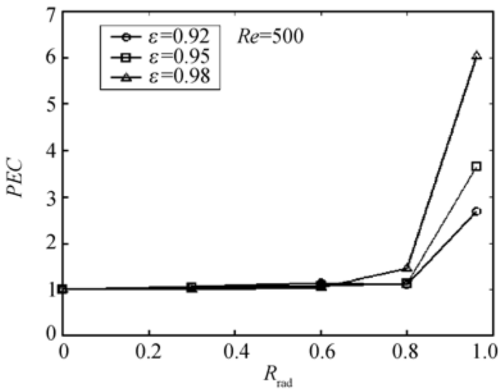

Figure 10 Increase of *PEC* value with porous radius ratio for air.

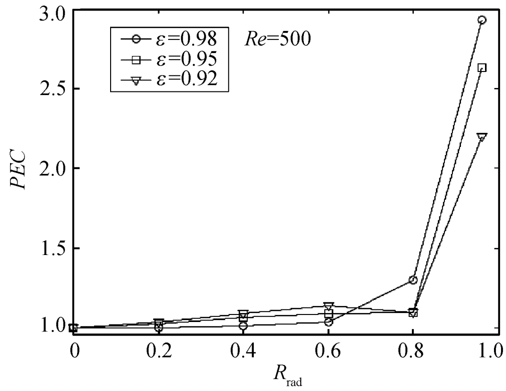

Figure 11 Increase of *PEC* value with porous radius ratio for water.

example, wire matrix and wire screen with high mesh number are such kind of typical porous media. When they are filled into the heat exchange tube of air cooler in petrochemical refining devices, the problems such as ash deposition and encrusted layer will not exist.

3 Field synergy analysis

According to the principle of field synergy[8,9] between fluid flow and heat transport by integrating the energy equation over a cross section in the fully developed tube flow, we can get

$$\frac{1}{R}\int_0^R r\rho c_p(\boldsymbol{U}\cdot\nabla T)\mathrm{d}r = k_f\left.\frac{\partial T}{\partial r}\right|_w = q\,. \tag{9}$$

Eq. (9) can be re-written as the following non-dimensional equation:

$$Nu = RePr\int_0^1 \bar{r}(\bar{\boldsymbol{U}}\cdot\nabla\bar{T})\mathrm{d}\bar{r}, \tag{10}$$

where $\bar{\boldsymbol{U}}$ represents non-dimensional velocity vector, and $\nabla\bar{T}$ represents non-dimensional temperature gradient.

The synergy angle α between fluid-velocity vector and temperature-gradient vector in the flow field can be expressed as

$$\cos\alpha = \frac{\boldsymbol{U}\cdot\nabla T}{|\boldsymbol{U}|\cdot|\nabla T|}\,. \tag{11}$$

Seen from eqs. (10) and (11), if the synergy angle α increase, $\boldsymbol{U}\cdot\nabla T$ will increase, and then *Nu* number will increase. Therefore, the synergy angle α can be used to evaluate the effects of heat transfer enhancement.

The local synergy angle distributions in the fully developed tube flow at different R_{rad} values for air and water as plotted in Figures 12 and 13 respectively. Compared with the tube without porous material, the local synergy angle in the region with porous material decreases notably, which raises the level of temperature uniformity, and increases the *Nu* number, thereby enhances heat transfer. As shown in the figures, the average synergy angle decreases with the porous radius ratio. Meanwhile, because the thermal conductivity and *Pr* number of air are smaller, the effect of heat transfer enhancement in the core flow for air is better than that for water.

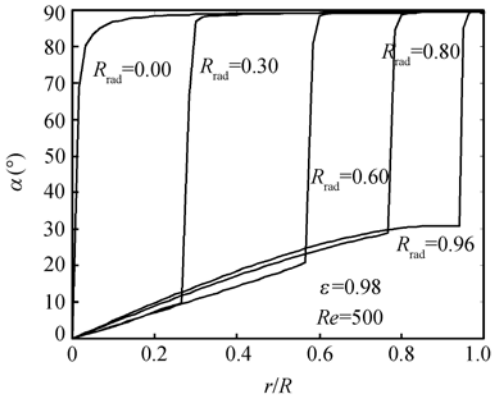

Figure 12 Variation of local synergy angle in the fully developed flow (water).

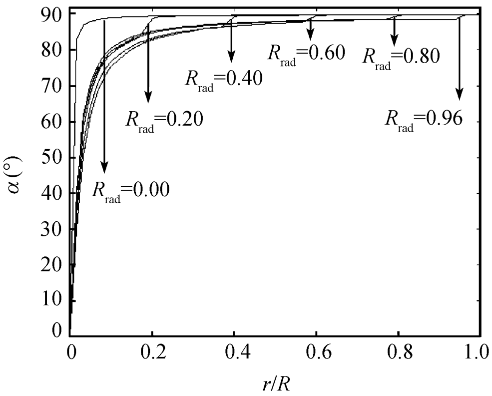

Figure 13 Variation of local synergy angle in the fully developed flow (water).

4 Conclusion

We have developed the theory and method of heat transfer enhancement in the core flow along a tube, which can be applied to the design of low *Pr* number fluid and laminar flow heat exchanger, and also can be recommended to some other application cases, such as low *Pr* number fluid and laminar flow in the tube bundle. To achieve the core flow enhancement inside a tube, we should follow several basic principles which make the core flow temperature uniform, reduce momentum dissipation and friction loss of boundary flow; and reduce fluid velocity gradient. The numerical calculation in this paper indicates that the porous media filled in the tube should be high thermal conductivity, high porosity and high porous radius ratio. When thermal conductivity, porosity and porous radius ratio of the porous material are 200 W/(m·℃), 0.92 and 0.96 respectively, the *PEC* values are close to 3 and 2.5 for air and water respectively. According to the methods proposed in this paper, we can also add some other inserts which are similar to porous media in the core flow of a tube to achieve heat transfer enhancement.

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