THERMAL NON-EQUILIBRIUM MODELING OF COUPLED HEAT AND MASS TRANSFER IN BULK ADSORPTION SYSTEM OF POROUS MEDIA

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According to the volume-averaging method, a two-dimensional thermal non-equilibrium mathematical model was developed to simulate the coupled heat and mass transfer in porous media with strong adsorption, in which local fluid and solid temperatures were dealt with separately. The temperature-dependent Langmuir isotherm was applied to describe the equilibrium characteristics of binary gas CO₂ adsorbed by zeolite 13X. The adsorption-generated heat was considered in the energy equation of the solid matrix. The coupling of two energy equations was made by considering the interfacial heat transfer term. The adsorption feature of a packed bed filled with porous adsorbent and the effects of bulk flow velocity, particle diameter, effective thermal conductivity of solids, and specific heat of adsorbent on product concentration were investigated numerically. The numerical simulation shows that the heat transfer characteristic has notable effects on the quantity of adsorption and the purity of the product. High adsorption efficiency can be achieved by improving the adsorbent physical properties and reducing the temperature span of the pressure swing adsorption process. Experimental data reported by other researchers validate the accuracy of the present model.

KEY WORDS: heat and mass transfer, non-equilibrium, porous media, pressure swing adsorption, carbon dioxide

1. INTRODUCTION

Pressure swing adsorption (PSA) technology based on the adsorption characteristics of the components has been widely applied to separate gas mixtures. Recently, the PSA process with strong adsorption has attracted special attention in separating carbon dioxide, one of the most predominant greenhouse gases, from flue gas. This process is characterized by several unique features. First of all, mass transfer plays an important role in the periodic adsorption process under a rapid cycle condition. Second, the operating temperature span caused by adsorption or release of adsorption-generated heat is commonly more than 10°C which leads to a significant change in adsorbed quantity. Finally, heat and mass transfer are coupled with each other. Therefore, it is necessary to consider both heat and mass transfer processes in analysis of the adsorber, especially in some large-scale PSA equipment (Steinbery, 1992; Kikkinides et al., 1993).

Heat and mass transfer in porous media have been investigated for several decades (Nield and Bejan, 1999). The local thermal equilibrium model (Cheng, 1978; Whitaker, 1986; Kim and Vafai, 1989; Rees and Vafai, 1999) and the local thermal non-equilibrium model (Quintard and Whitaker, 1993; Amiri and Vafai, 1998; Kim et al. 2000) have been used in dealing with heat transport in porous media. The former is relatively simple However it is valid only when the temperature difference between solid and fluid phases is very small The latter is relatively complex, as heat exchange between solid and fluid phases has to be considered. According to the criterion proposed by Kim and Jang (2002), the thermal non-
equilibrium model should be used to describe the strong adsorption processes in the porous packed bed.

The simplest and most commonly applied mass transfer model is the linear driving force (LDF) model, which was first proposed by Glueckauf and Coates (1947). The LDF model was derived by assuming that the driving force of mass transfer within a chromatographic process takes on a simple linear form between the compositions in equilibrium with the bulk phase (Carta, 1993; Hsuen, 2000) Vel dsink et al. (1995) adopted the Fick model and the dusty-gas model to the study mass transport process accompanied with the chemical reaction in a porous catalyst.

However, attention has not been fully paid to the effect of heat transfer on mass transfer for studying the coupling of heat and mass transfer in porous media subjected to chemical reactions or physical adsorption So, a mathematical model describing the coupled process of bulk adsorption has been established in this paper. Both the Forchheimer and Brinkman effects have been taken into account in the momentum equation. The thermal non-equilibrium model has been adopted and two energy equations have been coupled with the interfacial heat transfer term, in which the adsorption-generated heat has been considered in the energy equation of the solid matrix. The temperature-dependent Langmuir isotherm has been used to describe the equilibrium characteristics of CO₂ and N₂, and the mass transfer coefficient has been approximated as the external mass transfer coefficient of the particle. The adsorption features of the packed bed also have been investigated numerically.

2. MATHEMATICAL MODELING

2.1 Problem Description

The schematic diagram of the physical model is shown in Fig. 1. The cylindrical bed with a 257 mm diameter and a length of 1000 mm was filled with a homogenous and isotropic spherical adsorbent subjected to a constant tem-
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FIG. 1: Schematic diagram of a packed bed filled with zeolite 13X

perature boundary condition. The fluid was injected at a constant rate with uniform velocity $u_0$. Due to the symmetry of the packed bed, only half of it was considered in the numerical simulation.

The following assumptions are made in the modeling:

1. Ideal gas law is applicable.
2. The temperature of a particle is uniform.
3. Natural convection and radiation heat transfer are neglected.
4. The physical properties of the gas phase, adsorbent and column wall are constant.
5. The heat capacity of the adsorbed gas is neglected.

Under the above assumptions, the governing equations are obtained as follows:

1. For overall mass balance:
   \[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{1}{r} \frac{\partial (r \rho v)}{\partial r} + \rho_b \sum_{k=1}^{n} \frac{\partial q_k}{\partial t} = 0 \] (1)

2. For mass balance for gas $i$ (CO$_2$ or N$_2$):
   \[ \frac{\partial \rho p i}{\partial t} + \frac{\partial \rho u}{\partial x} (\rho u i) + \frac{1}{r} \frac{\partial (r \rho v i)}{\partial r} - \frac{\partial}{\partial x} \left( D_b \frac{\partial C_i}{\partial x} \right) - \frac{1}{r} \frac{\partial}{\partial r} \left( r D_b \frac{\partial C_i}{\partial r} \right) + \rho_b \frac{\partial q_i}{\partial t} = 0 \] (2)

3. For the Darcy–Forchheimer–Brinkman momentum equation (Sozen and Kuzay, 1996):
   \[ \rho_f \left( \frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot \nabla \bar{V} \right) = -\nabla p + \mu_{\text{eff}} \nabla^2 \bar{V} - \frac{\mu_f}{K} \bar{V} - \rho_f \frac{F \rho}{\sqrt{K}} |\bar{V}| \bar{V} \] (3)

The second, third and fourth terms on the righthand side of Eq. (3) represent the viscous effect, frictional resistance and inertial force respectively.

The permeability of the porous bed (Ergun, 1952) and geometric factor $F$ in the momentum equation are given as (Quintard and Whitaker, 1993)

\[ K = \frac{\rho^3 d^2}{150 (1 - \rho)^2} \] (4)

\[ F = \frac{1.75}{150 \rho^{3/2}} \] (5)

The energy conservation equations for the fluid and solid matrix are given as

\[ \frac{\partial [\rho C_f \rho T_f]}{\partial t} + \frac{\partial (\rho u C_f T_f)}{\partial x} + \frac{1}{r} \frac{\partial (r \rho v C_f T_f)}{\partial r} = \frac{\partial}{\partial x} \left( \lambda_{f,\text{eff}} \frac{\partial T_f}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_{f,\text{eff}} \frac{\partial T_f}{\partial r} \right) + h_v (T_s - T_f) \] (6)

\[ \frac{\partial [C_s \rho_b T_s]}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_{s,\text{eff}} \frac{\partial T_s}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_{s,\text{eff}} \frac{\partial T_s}{\partial r} \right) - h_v (T_s - T_f) \sum_{k=1}^{n} H_k \rho_b \frac{\partial q_k}{\partial t} \] (7)

The mass transfer is described by the LDF model,

\[ \rho_b \frac{\partial q_i}{\partial t} = h_{m,v} (C_i - C_i^*) \] (8)

The fluid and solid effective thermal conductivities are

\[ \lambda_{f,\text{eff}} = \varphi \lambda_f \] (9)

\[ \lambda_{s,\text{eff}} = (1 - \varphi) \lambda_s \] (10)

The volumetric solid-to-fluid heat and mass transfer coefficients $h_v$ and $h_{m,v}$ are

\[ h_v = 6h_{sf} (1 - \varphi)/d_p \] (11)

\[ h_{m,v} = 6h_{mf} (1 - \varphi)/d_p \] (12)
where heat transfer coefficient \( h_{sf} \) is given by (Wakao et al., 1979)

\[
\text{Nu} = \frac{h_{sf}d_p}{\lambda_f} = 2.0 + 1.1\text{Pr}^{1/3}\text{Re}_p^{0.6}
\]

\[
\text{Pr} = \frac{C_p\mu}{k}, \quad \text{Re}_p = \frac{\rho Ud_p}{\mu}
\]

where Reynolds number \( \text{Re}_p \) covers the range up to 8500.

Mass transfer coefficient \( h_{m, st} \) is given by

\[
\text{Sh} = \frac{D_lh_{m, st}}{D_m} = 2.0 + 0.6\text{Sc}^{1/3}\text{Re}^{1/2}
\]

\[
\text{Sc} = \frac{\mu}{\rho fD_m}, \quad \text{Re} = \frac{\rho fV\varphi d_f}{\mu}
\]

The temperature-dependent Langmuir isotherm (Harlick and Tezel, 2001) is used to represent the gas–solid equilibrium isotherm:

\[
q_i = \frac{q_{\text{max},i}b_i pC_i^*}{1 + \sum_{k=1} q_{\text{max},i}b_k pC_k^*}
\]

where \( q_{\text{max},i} = B_{1,i} \exp (B_{2,i}T) \) and \( b_i = B_{3,i} \exp (B_{4,i}T) \).

The parameter values used in the simulation are listed in Table 1, in which the Langmuir parameters for CO\(_2\) and N\(_2\) were measured by the constant volume method at different pressures (Suzuki, 1990).

The initial and boundary conditions are given as follows:

1. Initial conditions (t = 0):

\[
U = U_0, \quad V = 0
\]

\[
T_f = T_s = T_w = 330\text{K}
\]

\[
C_{\text{CO}_2} = 0, \quad C_{\text{N}_2} = 1.0
\]

\[
q_{\text{CO}_2} = 0, \quad q_{\text{N}_2} = q_{\text{N}_2}^* (C_{\text{N}_2} = 1.0)
\]

2. Boundary conditions (t > 0):

\[
x = 0 : \quad U = U_0, \quad V = 0, \quad C_{\text{CO}_2} = 0.1
\]

\[
C_{\text{N}_2} = 0.9, \quad T_f = T_s = T_w = 330\text{K}
\]

\[
r = 0 : \quad \frac{\partial U}{\partial r} = 0, \quad \frac{\partial V}{\partial r} = 0, \quad \frac{\partial C_{\text{CO}_2}}{\partial r} = 0,
\]

\[
\frac{\partial C_{\text{N}_2}}{\partial r} = 0, \quad \frac{\partial T_s}{\partial r} = 0, \quad \frac{\partial T_f}{\partial r} = 0
\]

\[
r = R : \quad U = 0, \quad V = 0, \quad \frac{\partial C_{\text{CO}_2}}{\partial r} = 0,
\]

\[
\frac{\partial C_{\text{N}_2}}{\partial r} = 0.0, \quad T_f = T_s = T_w = 330\text{K}
\]

### 3. RESULTS AND DISCUSSIONS

The numerical solution is obtained by employing the finite-volume method. The convective flux term is discretized by the second-order central differencing scheme that is blended with the upwind differencing scheme using the deferred correction method. The SIMPLE algorithm for pressure–velocity coupling is applied. The source term of the equation is linearized in such a way that ensures diagonal dominance in the corresponding discretized equation (Ferziger and Peric, 1999). The discrete equations cannot be solved independently because of the cross-coupling between mass and energy balance equations. Therefore, the iterative method was adopted to solve the temperature and concentration fields. Convergence is checked in terms of the maximum change (10\(^{-4}\)) for every variable during the iteration at each time increment. All computations have been carried out for a half-cylindrical reactor with non-uniform grid arrangements. Different grid sizes, such as 50 \(\times\) 25 100 \(\times\) 50, 200 \(\times\) 100, 400 \(\times\) 200, and 800 \(\times\) 400 have been used in the domain of \(L \times R\). As the computational accuracy by taking grid sizes of 200 \(\times\) 100, 400 \(\times\) 200, and 800 \(\times\) 400 are almost the same, we chose the grid size of 200 \(\times\) 100 in the computations.

In order to verify the present model, numerical simulation was performed under the same conditions as those of the experiment conducted by Hwang et al. (1995). As shown in Fig. 2, a comparison of outlet CO\(_2\) concentration was made between the numerical solutions and experimental data, in which agreement between the theoretical calculation and experimental measurement can be observed, which validates the present model to some extent.

The four main parameters impacting an adsorption process, such as bulk flow velocity, particle diameter, ef-

### TABLE 1: Parameter values in simulation of temperature-dependent Langmuir isotherm

<table>
<thead>
<tr>
<th>Langmuir parameters</th>
<th>CO(_2)</th>
<th>N(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_1) (kg/kg)</td>
<td>2.2</td>
<td>0.28</td>
</tr>
<tr>
<td>(B_2) (1/T)</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>(B_3) (1/atm)</td>
<td>500</td>
<td>60</td>
</tr>
<tr>
<td>(B_4) (1/T)</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
FIG. 2: Comparison between outlet CO$_2$ concentrations obtained by the present model and the experiment conducted by Hwang et al. (1995)

3.1 Effect of Bulk Flow Velocity

The influences of the bulk flow velocity on the transport and adsorption characteristics are shown in Figs. 3–6. In Fig. 3 it can be seen that the pressure drop in the packed bed filled with adsorbent increases with the increase of

FIG. 3: Effect of bulk flow velocity on the pressure drop between the inlet and outlet

FIG. 4: Effect of bulk flow velocity on CO$_2$ concentration at the outlet

FIG. 5: Effect of bulk flow velocity on outlet fluid temperature

FIG. 6: Effect of bulk flow velocity on average adsorbent temperature
bulk flow velocity, which indicates that more energy is consumed. Figure 4 shows that the larger the bulk flow velocity, the shorter the adsorption time will be. Thus the transition point of the breakthrough curve appears earlier. Therefore, increasing the bulk flow velocity can change the capacity of per-volume adsorbent. Figures 5 and 6 demonstrate that the outlet fluid temperature and average adsorbent temperature increase more rapidly with the increase of bulk flow velocity in the upgoing stage of curves. However, the maximum outlet fluid temperature decreases with an increase of the bulk flow velocity, while this tendency is inversed for the maximum average adsorbent temperature. The reasons for those phenomena can be explained as the increase of the average adsorbent temperature being dependent on the release rate of adsorption heat, the heat transfer rate between solid and fluid and the heat capacity of the solid adsorbent while the increase of bulk fluid temperature depends on the heat transfer rate between solid and fluid and the heat capacity of fluid. If the bulk flow velocity is higher, the adsorption rate will increase, and adsorption-generated heat will release more quickly, which leads to a quicker increase in the solid adsorbent temperature. Meanwhile, a higher bulk flow velocity results in a relatively shorter time for fluid residence. Thus heat exchange between fluid and adsorbent is not sufficient, which leads to a lower outlet fluid temperature.

3.2 Effect of Particle Diameter

Figures 7–10 display the effect of the particle diameter of adsorbent on the transport and adsorption characteristics of the packed bed. From Fig. 7 it can be seen that pressure

FIG. 7: Effect of particle diameter on the pressure drop between the inlet and outlet

FIG. 8: Effect of particle diameter on outlet CO₂ concentration

FIG. 9: Effect of particle diameter on outlet fluid temperature

FIG. 10: Effect of particle diameter on average adsorbent temperature
drop decreases greatly with an increase of the particle diameter. It is expected that the permeability of the packed bed increases significantly when the particle diameter becomes larger. This indicates that adopting bigger adsorbent particles can save more energy. Figure 8 shows that the larger the particle diameter, the higher the CO$_2$ concentration in the outflow will be. It is noted from Fig. 9 that the outlet fluid temperature increases more slowly, as the particle diameter increases, while this tendency is inverted for the average adsorbent temperature shown in Fig. 10. It is expected that when other conditions are fixed, the heat and mass transfer coefficients decrease with the increase of the particle diameter. In the case of larger particles, the increase of average adsorbent temperature leads to a lower equilibrium adsorption quantity. Consequently the adsorption performance will be slightly worse.

3.3 Effect of Effective Solid Thermal Conductivity

As shown in Fig. 11, the time it takes until the appearance of CO$_2$ in the outflow is almost the same for different curves. After that, however, the outlet CO$_2$ concentration increases, and its increase rate is relatively slower when $\lambda_{s,\text{eff}}$ is larger. As shown in Figs. 12 and 13, for both the outlet fluid temperature and the average adsorbent temperature the increase rates are also relatively slower when $\lambda_{s,\text{eff}}$ is larger. It can be explained as follows: when effective solid thermal conductivity $\lambda_{s,\text{eff}}$ is raised, more adsorption heat is transferred to the surroundings, which makes the average adsorbent temperature increase more slowly and the equilibrium adsorption quantity increase.

As a result, the adsorption performance of the packed bed will be improved.

3.4 Effect of Solid Specific Heat

Figures 14–16 demonstrate that when the solid specific heat is larger it takes more time until the appearance of CO$_2$ in the outflow, and the outlet CO$_2$ concentration, outlet fluid temperature and average adsorbent temperature increase more slowly. These phenomena can be explained...
as follows: when the same amount of heat is absorbed, higher solid specific heat means larger heat capacity of the adsorbent which results in a smaller temperature swing. Thus, according to the temperature-dependent Langmuir isotherm, the equilibrium adsorption quantity increases. Therefore, more gas mixtures can be separated and the purity of the production can be raised by increasing the solid specific heat.

4. CONCLUSIONS

A local thermal non-equilibrium mathematical model describing the coupled heat and mass transfer in a porous adsorbent bed was established. The model was solved by the finite-volume method and verified through the comparison between simulation results and experimental data reported by other researchers. The effects of the bulk flow velocity, particle diameter, effective solid thermal conductivity and specific heat of the adsorbent were investigated numerically. The computation results show that (1) increasing the bulk flow velocity can enhance the capacity of per-volume adsorbent, but worsen the heat transfer performance and consume more energy; (2) increasing the particle diameter can slightly worsen heat and mass transfer performance, but flow resistance decreases significantly; (3) adsorption performance can be slightly improved when the effective solid thermal conductivity becomes larger; and (4) adsorption performance can be significantly improved when the solid specific heat is increased, while energy consumption remains almost the same. Those results may provide some useful information for operating a PSA process.

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