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Performance of micro two-level heat devices with prior information

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The performance of micro two-level heat engines and refrigerators with prior information has been analyzed under the maximum power output and maximum χ figure of merit, respectively. Under the asymmetric limits, the Curzon–Ahlborn efficiency and Curzon–Ahlborn coefficient of performance are retrieved. However, they are independent of the probability distribution function of particle numbers. Furthermore, the results are in accord with previous literatures. They shed light on that the model proposed in this paper can describe any specified models with concrete prior probability distribution such as two-level quantum heat devices and Brownian heat devices with prior information.

1. Introduction

The urgency for energy saving and fuel depletion has attracted rising attention for the optimization of real thermodynamical cycles. In classic thermodynamics, Carnot efficiency η_C and Carnot COP ε_C have defined the maximum energy conversion rate for heat convertors operating between two heat reservoirs at temperatures $T_1 > T_2$ [1]. However the realization of η_C and ε_C leads to vanishing power extracted for heat engines and zero cooling load rate for refrigerators, since they are reached only in reversible cycles where all the processes are quasi-static, and the cycle time durations are infinite. The ideal Carnot cycles must be sped up to meet the actual demand.

Considering finite time durations of the heat transfer processes between heat reservoirs and working fluid, Curzon–Ahlborn [2] proposed the concept of endoreversible Carnot heat engine, and deduced its efficiency at maximum power (MP) output. That is the groundbreaking CA efficiency $\eta_{\mathrm{CA}} = 1 - \sqrt{T_c/T_h}$. It lay the foundation for finite time thermodynamics. Many revisions of the CA model have been made to describe the real-life heat engines more accurately, and some good results at the maximum power output criterion have been obtained [3–7]. Furthermore, the low dissipation model [8–11] and linear and minimally nonlinear irreversible heat engine models described by the Onsager relations and the extended Onsager relations [12–14] have been also proposed to study the efficiency and its bounds at MP criterion. In these models, the CA efficiency is recovered under the symmetric conditions. In addition, the efficiency at the MP criterion for stochastic Brownian heat engines [15] and the Feynman ratchet heat engine [16] have been studied further.

However for refrigerators, the minimum power input is not an appropriate optimization criterion [17], and much research has been dedicated to selecting figure of merits for optimizing refrigerators. by maximizing the per-unit-time COP, Velasco et al. [18] obtained the upper bound of COP, $\varepsilon_{\mathrm{CA}} = \sqrt{\varepsilon_C + 1} - 1$, i.e. the CA coefficient of performance, for endoreversible refrigerators with $\varepsilon_C = T_c/(T_h - T_c)$ being the Carnot COP, where T_c and T_h are the temperatures of the cold and hot reservoirs, respectively. In addition, Yan and Chen [19] conducted the optimization with the objective function $\varepsilon \dot{Q}_c$ where $\dot{Q}_c$ is the cooling load rate of the refrigerators. To step further, de Tomás et al. [20] introduced the unified optimization criterion χ both for heat engines and refrigerators. By taking χ as the objective function, based on the low dissipation model, Wang et al. [17] proposed that the COP at maximum χ was bounded between 0 and $(\sqrt{9 + 8\varepsilon_C} - 3)/2$. Besides, through the minimally nonlinear irreversible refrigerator model, Y. Izumida et al. [21] also obtained the same bounds as those in Ref. [17] under the tight coupling condition. Long et al. [22] studied general refrigerators with non-isothermal processes and also derived the CA coefficient of performance, which was independent of the time duration completing either process. Some new bounds were also reported in their later work [23]. In addition, Allahverdyan et al. [24] also investigated quantum refrigerators and obtained some new bounds of COP under the χ figure of merit.

Furthermore, another figure of merit Ω, accounting for both the energy benefits and losses was proposed by Hernández et al. to analyze heat converters [25]. Based on the Ω criterion, de Tomas et al. [26] and Long et al. [27] obtained the efficiency and COP bounds

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for heat engines and refrigerators, respectively, through the low dissipation model and the minimally nonlinear irreversible model. The efficiencies for the stochastic heat engine cycle model and the nanothermoelectric engine mode have been also studied [18], The COP of low dissipation refrigerators with irreversibility in the adiabatic processes was also considered by Hu et al. [28] under the Ω criterion. Furthermore, the performance of general heat devices with non-isothermal processes under the Ω criterion were also analyzed [29].

To investigate a system, we may possess some prior information about a parameter that is uncertain, however, the possible range of values are known. Furthermore, we can assign probabilities for the likely values of this parameter. That is the prior information, based on which we can make estimates about the behavior of the system. Recently prior probability has been adopted to performance of the quantum and traditional heat engines under the MP criterion [30–33]. And the CA efficiency has been also obtained in the asymmetric limits. It provides a new way for optimizing heat devices. In this paper, we first introduce general micro two-level heat engine model with prior information in Section 2. Then the efficiency under the MP criterion has been studied. The general constraint function of the efficiency has been deduced. Under the asymmetric limits, the CA efficiency is retrieved, which is accord with previous literatures. In Section 3, the COP of general refrigerators under the χ criterion has been also studied. Under the asymmetric limits, the CA coefficient of performance is also recovered. Finally some concluding remarks are given.

2. Micro heat engines

Consider a two-level micro system, the particles are coupled with two heat baths at temperatures T_1 and T_2 ($T_1 > T_2$). The energy potential of the particles coupled to the hot and cold baths are ϕ_1 and ϕ_2, respectively. They are in thermodynamics equilibrium with the coupled baths. The probability of the particle numbers are assumed to be $f(x)$, where $x = \phi_i/K_B T_i$ ($i = 1, 2$) and K_B is the Boltzmann constant. This assumption is in accord with the Bose–Einstein statistics, the Fermi–Dirac statistics in the quantum thermodynamics, Maxwell–Boltzmann statistics and the Brownian systems in the statistical thermodynamics [34–42]. All of the distribution functions share the exponent item $\exp(\phi_i/K_B T_i)$. For a Feynman's ratchet as a heat engine, the particle moves forward and backward. The particle is in contact with different reservoirs at different positions. $f(x)$ is the forward/backward probability. For the two-level quantum system, $f(x)$ is the occupation probability of the quantum particles which contacts with the heat reservoirs. In the initial state, the energy of the particles in contact with the hot reservoir is $E_{h,int} = \phi_1 f(\phi_1/T_1)$; in the final state, the two systems swap between themselves their initial probability distributions. The final energy of those particles at the end of the work-extracting transformation is given by $E_{h,fin} = \phi_1 f(\phi_2/T_2)$. Therefore the heat absorbed can be expressed as

$$\dot{Q}_h = E_{h,int} - E_{h,fin} = \phi_1 R\big(f(\phi_1/T_1) - f(\phi_2/T_2)\big) \tag{1}$$

where R is the rate constant with the dimension $1/s$. Similarly, power extracted reads

$$P = \phi_1 R\big(f(\phi_1/T_1) - f(\phi_2/T_2)\big) - \phi_2 R\big(f(\phi_1/T_1) - f(\phi_2/T_2)\big) \tag{2}$$

For a Feynman's ratchet as a heat engine, the particle moves forward and backward. The particle is in contact with different reservoirs at different positions. There also exists a swap operation. The same expressions for heat absorbed or released also hold. The efficiency of the system is $\eta = (\phi_1 - \phi_2)/\phi_1$. To start with, we assume the efficiencyη is prescribed, but the exact values of ϕ_1 and ϕ_2 are not clear. Therefore, the values of ϕ_1 and ϕ_2 are dependent on each other. That is to say, for particles with ϕ_1, there should exist another coupled ones with ϕ_2. Hence the power output can be written as a function of either η and ϕ_2 or η and ϕ_1. Here we rewrite the power as a function of η and ϕ_2, thus

$$P(\eta,\phi_2) = \frac{\phi_2 \eta R_0}{1-\eta}\left(f\left(\frac{\phi_2}{(1-\eta)K_B T_1}\right) - f\left(\frac{\phi_2}{K_B T_2}\right)\right) \tag{3}$$

Furthermore, the only prior information about the parameter ϕ_2 is that it takes positive real values in the range $\phi_2 \in [\phi_2^{\min}, \phi_2^{\max}]$, where $\phi_2^{\max}$ and $\phi_2^{\min}$ are respectively the upper and lower bounds of the particle energy potentials for ϕ_2. Here $\Gamma(\phi_2)$ denotes the prior distribution of particle energy potentials. Now the expected value of power is given by

$$\overline{P}(\eta) = \int_{\phi_2^{\min}}^{\phi_2^{\max}} \frac{\phi_2 \eta R_0}{1-\eta}\left(f\left(\frac{\phi_2}{(1-\eta)K_B T_1}\right) - f\left(\frac{\phi_2}{K_B T_2}\right)\right)\Gamma(\phi_2)d\phi_2 \tag{4}$$

The value of $\overline{P}(\eta)$ strongly depends on the prior energy potential distribution $\Gamma(\phi_2)$. Therefore, the choosing of the prior distribution plays essential roles in the power output. Based on the Bayesian approach, Thomas and Johal [31] derived that the distribution of the particles with energy ϕ_2, that is $\Gamma(\phi_2) = 1/[\phi_2 \ln(\phi_2^{\max}/\phi_2^{\min})]$, reflecting that the particles convey lower energy much easier than the higher. This distribution has been adopted to investigate the performance of different heat engines [31,33]. Thereby Eq. (4) yields to

$$\begin{aligned}\overline{P}(\eta) &= \frac{\eta R_0}{(1-\eta)\ln(\phi_2^{\max}/\phi_2^{\min})}\int_{\phi_2^{\min}}^{\phi_2^{\max}}\left(f\left(\frac{\phi_2}{(1-\eta)K_B T_1}\right)\right. \\ &\qquad \left. - f\left(\frac{\phi_2}{K_B T_2}\right)\right)d\phi_2 \\ &= \frac{\eta R_0 K_B}{(1-\eta)\ln(\phi_2^{\max}/\phi_2^{\min})}\big(A(1-\eta)T_1 - BT_2\big)\end{aligned} \tag{5}$$

where

$$A = \int_{\phi_2^{\min}/(1-\eta)K_B T_1}^{\phi_2^{\max}/(1-\eta)K_B T_1} f(x)\,dx \tag{6}$$

and

$$B = \int_{\phi_2^{\min}/K_B T_2}^{\phi_2^{\max}/K_B T_2} f(x)\,dx \tag{7}$$

According to Eq. (6) and Eq. (7), B is independent of η, while generally A is not. Therefore, we can get the optimal efficiency ($\eta_{\overline{P}}$) leading to the maximum power $\overline{P}(\eta)$ by letting $\partial\overline{P}(\eta)/\partial\eta = 0$. Thus we have

$$\begin{aligned}AT_1 &+ \frac{\eta}{K_B(1-\eta)^2}\left[\phi_2^{\max} f\left(\frac{\phi_2^{\max}}{(1-\eta)K_B T_1}\right)\right. \\ &\left. - \phi_2^{\min} f\left(\frac{\phi_2^{\min}}{(1-\eta)K_B T_1}\right)\right] - \frac{BT_2}{(1-\eta)^2} = 0\end{aligned} \tag{8}$$

Eq. (8) is the general constraint function of the efficiency for micro heat engines with prior information under the maximum

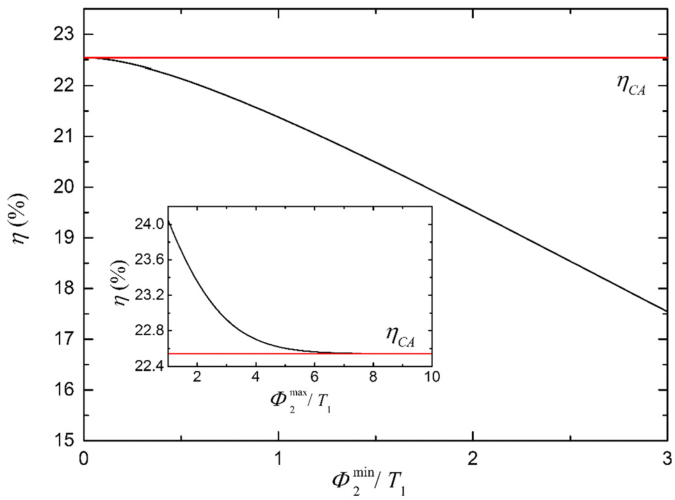

Fig. 1. Efficiency at maximum expected power is plotted versus $\phi_2^{\min}$ (scaled by T_1) where $\phi_2^{\max}/T_1 = 20$, $T_1 = 500$ K and $T_2 = 300$ K. The efficiency is also plotted versus $\phi_2^{\max}$, with $\phi_2^{\min}/T_1 = 0$. For lower values of $\phi_2^{\min}$ and higher values of $\phi_2^{\max}$, the efficiency approaches the CA efficiency.

power output. The efficiency depends on the probability distribution function of the particle numbers and the scales of energy potentials. If they are specified, then the efficiency at the maximum power output can be calculated. Furthermore, provided $\phi_2^{\min}/K_BT_2 \to 0$ and $\phi_2^{\max}/K_BT_2 \to \infty$, based on the relation between ϕ_1 and ϕ_2, we have $\phi_i^{\min}/K_BT_i \to 0$ and $\phi_i^{\max}/K_BT_i \to \infty$ $(i = 1, 2)$, and $A = B$. Thereby Eq. (8) is reduced to

$$T_1 - \frac{T_2}{(1-\eta)^2} = 0 \tag{9}$$

The solver of Eq. (9) reads

$$\eta_{\bar{P}} = 1 - \sqrt{T_2/T_1} \equiv \eta_{CA} \tag{10}$$

That is the CA efficiency. It has been obtained through the endoreversible Carnot model, linear irreversible model, low dissipation model and minimally nonlinear heat engine model, and quantum heat engines under the symmetric conditions [8–11,43]. Furthermore, it is independent of the probability distribution function of particle numbers. Therefore, the results obtained in this paper could describe any specified models with concrete prior probability distribution.

For example, when the distribute function is given by $f(x) = \exp(-\phi_i/K_BT_i)$, the Feynman's ratchet heat engine is retrieved. Eq. (8) can be rewritten as

$$\begin{aligned} &T_1\left(e^{-\phi_2^{\min}/(1-\eta)T_1} - e^{-\phi_2^{\max}/(1-\eta)T_1}\right) \\ &\quad + \frac{\eta}{(1-\eta)^2}\left[\phi_2^{\max}e^{-\phi_2^{\max}/(1-\eta)T_1} - \phi_2^{\min}e^{-\phi_2^{\min}/(1-\eta)T_1}\right] \\ &\quad - \frac{T_2}{(1-\eta)^2}\left(e^{-\phi_2^{\min}/T_2} - e^{-\phi_2^{\max}/T_2}\right) = 0 \end{aligned} \tag{11}$$

Here we set $K_B = 1$. According to Eq. (11), we can generate the relation of the expected efficiency at maximum power with $\phi_2^{\min}$ and $\phi_2^{\max}$, which is present in Fig. 1. The CA efficiency is also obtained provided $\phi_i^{\max}/K_BT_i \to \infty$ and $\phi_i^{\min}/K_BT_i \to 0$. In Refs. [30,31], Johal et al. have studied a quantum heat cycle in the quantum systems, the occupation probabilities are $f(x) = 1/(\exp(\phi_i/K_BT_i) + 1)$. They found that the CA efficiency is also recovered under the asymmetric limits.

3. Micro refrigerators

Similarly to heat engines, for a micro refrigerator, the COP can be calculated as $\varepsilon = \phi_2/(\phi_1 - \phi_2)$. And the χ figure of merit can be expressed as

$$\chi = \varepsilon\phi_2 R\big(f(\phi_2/K_BT_2) - f(\phi_1/K_BT_1)\big) \tag{12}$$

Now consider a situation in which the energy scales ϕ_1 and ϕ_2 are only given by a prior distribution which a prescribed efficiency ε. Therefore with $\varepsilon = \phi_2/(\phi_1 - \phi_2)$, Eq. (12) can be rewritten as a function of ϕ_2, that is

$$\chi(\varepsilon,\phi_2) = \varepsilon\phi_2 R\left(f(\phi_2/K_BT_2) - f\left(\frac{\varepsilon+1}{\varepsilon}\phi_2/K_BT_1\right)\right) \tag{13}$$

The only prior information about the parameter ϕ_2 is that it takes positive real values in the range $\phi_2 \in [\phi_2^{\min}, \phi_2^{\max}]$. We assume the same prior $\Gamma(\phi_2)$ can also be applied. The expected value of χ is given by

$$\begin{aligned} &\bar{\chi}(\varepsilon) \\ &\quad = \int_{\phi_2^{\min}}^{\phi_2^{\max}} \varepsilon\phi_2 R\left(f(\phi_2/K_BT_2) - f\left(\frac{\varepsilon+1}{\varepsilon}\phi_2/K_BT_1\right)\right)\Gamma(\phi_2)d\phi_2 \\ &\quad = \frac{RK_B}{\ln(\phi_2^{\max}/\phi_2^{\min})}\left(B\varepsilon T_2 - \frac{A\varepsilon^2 T_1}{\varepsilon+1}\right) \end{aligned} \tag{14}$$

We can get optimal COP ($\varepsilon_{\bar{\chi}}$) leading to the maximum $\bar{\chi}(\varepsilon)$ by letting $\partial\bar{\chi}/\partial\varepsilon = 0$. Under the conditions $\phi_i^{\min}/K_BT_i \to 0$ and $\phi_i^{\max}/K_BT_i \to \infty$, we have

$$T_2 - \frac{2\varepsilon+\varepsilon^2}{(\varepsilon+1)^2}T_1 = 0 \tag{15}$$

The solver of Eq. (15) gives the COP at the maximum χ criterion

$$\varepsilon_{\bar{\chi}} = \sqrt{1+\varepsilon_C} - 1 \equiv \varepsilon_{CA} \tag{16}$$

That is the CA coefficient of performance, which has been obtained through the endoreversible Carnot model, linear irreversible model low dissipation model and minimally nonlinear model under the symmetric conditions [17,20,21]. It is also independent of the probability distribution function of particle numbers. Therefore, the results obtained in this paper could describe any specified micro refrigerators with concrete prior probability distribution. In Ref. [44], Feynman's ratchet refrigerator with prior information was studied, the backward and forward probabilities are $f(x) = \exp(-\phi_i/K_BT_i)$, the CA coefficient of performance is also retrieved provided $\phi_i^{\max}/K_BT_i \to \infty$ and $\phi_i^{\min}/K_BT_i \to 0$.

4. Conclusions

In this paper, the general micro heat engines and refrigerators with prior information have been analyzed under the maximum power output and maximum χ figure of merit, respectively. Jeffreys prior distribution has been adopted to the average power output for heat engines and χ figure of merit for refrigerators. And some important results have been obtained:

1. For heat engines, the constraint function of the efficiency at MP has been proposed. Under the asymmetric limits $\phi_i^{\max}/K_BT_i \to \infty$ and $\phi_i^{\min}/K_BT_i \to 0$, the CA efficiency is retrieved. And it is independent of the probability distribution function of particle numbers.

2. For refrigerators with prior information, the CA coefficient of performance is also obtained under the asymmetric limits. It does not depend on the probability distribution function of particle numbers.

The obtained results are also compared with those by the quantum and Brownian heat devices with prior information. It turns out that the model proposed in this paper can describe any specified models with concrete prior probability distribution. The average performance of micro heat engine/refrigerator models with prior information are equal to that of the traditional micro ones if the energy potentials obey the Jeffreys prior and minimum and maximum values of energy potentials fulfill the asymmetric limits. This paper could offer a more insightful perspective to study micro heat engines and refrigerators.

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References

[1] S. Carnot, Reflexions sur la Puissance Motrice du Feu et sur les Machines, Ecole Polytechnique, Paris, 1824.
[2] F.L. Curzon, B. Ahlborn, Am. J. Phys. 43 (1975) 22.
[3] A. Durmayaz, O.S. Sogut, B. Sahin, H. Yavuz, Prog. Energy Combust. Sci. 30 (2004) 175.
[4] L. Chen, F. Sun, C. Wu, J. Phys. D, Appl. Phys. 32 (1999) 99.
[5] Z. Yan, J. Chen, J. Chem. Phys. 92 (1990) 1994.
[6] P. Salamon, J. Chem. Phys. 74 (1981) 3546.
[7] J. Gonzalez-Ayala, L.A. Arias-Hernandez, F. Angulo-Brown, Phys. Rev. E 88 (2013) 052142.
[8] M. Esposito, R. Kawai, K. Lindenberg, C. Van den Broeck, Phys. Rev. Lett. 105 (2010) 150603.
[9] J. Wang, J. He, Phys. Rev. E 86 (2012) 051112.
[10] Y. Wang, Z.C. Tu, Phys. Rev. E 85 (2012) 011127.
[11] J. Guo, J. Wang, Y. Wang, J. Chen, Phys. Rev. E 87 (2013) 012133.
[12] C. Van den Broeck, Phys. Rev. Lett. 95 (2005) 190602.
[13] L. Onsager, Phys. Rev. 37 (1931) 405.
[14] Y. Izumida, K. Okuda, Europhys. Lett. 97 (2012) 10004.
[15] T. Schmiedl, U. Seifert, Europhys. Lett. 81 (2008) 20003.
[16] Z.C. Tu, J. Phys. A, Math. Theor. 41 (2008) 312003.
[17] Y. Wang, M. Li, Z.C. Tu, A.C. Hernández, J.M.M. Roco, Phys. Rev. E 86 (2012) 011127.
[18] S. Velasco, J.M.M. Roco, A. Medina, A.C. Hernández, Phys. Rev. Lett. 78 (1997) 3241.
[19] Z. Yan, J. Chen, J. Phys. D, Appl. Phys. 23 (1990) 136.
[20] C. de Tomás, A.C. Hernández, J.M.M. Roco, Phys. Rev. E 85 (2012) 010104(R).
[21] Y. Izumida, K. Okuda, A. Calvo Hernández, J.M.M. Roco, Europhys. Lett. 101 (2013) 10005.
[22] R. Long, W. Liu, J. Phys. A, Math. Theor. 47 (2014) 325002.
[23] R. Long, W. Liu, Phys. Scr. 90 (2015) 025207.
[24] A.E. Allahverdyan, K. Hovhannisyan, G. Mahler, Phys. Rev. E 81 (2010) 051129.
[25] A.C. Hernández, A. Medina, J. Roco, J. White, S. Velasco, Phys. Rev. E 63 (2001) 037102.
[26] C. de Tomas, J.M.M. Roco, A.C. Hernández, Y. Wang, Z.C. Tu, Phys. Rev. E 87 (2013) 011127.
[27] R. Long, Z. Liu, W. Liu, Phys. Rev. E 89 (2014) 062119.
[28] Y. Hu, F. Wu, Y. Ma, J. He, J. Wang, A. Hernández, J. Roco, Phys. Rev. E 88 (2013) 062115.
[29] R. Long, W. Liu, Phys. Rev. E 91 (2015) 042127.
[30] R.S. Johal, Phys. Rev. E 82 (2010) 061113.
[31] G. Thomas, R.S. Johal, Phys. Rev. E 85 (2012) 041146.
[32] G. Thomas, P. Aneja, R.S. Johal, Phys. Scr. T 151 (2012) 014031.
[33] P. Aneja, R.S. Johal, J. Phys. A, Math. Theor. 46 (2013) 365002.
[34] R.K. Niven, Phys. Lett. A 342 (2005) 286.
[35] R.F. Cracknell, D. Nicholson, N. Quirke, Phys. Rev. Lett. 74 (1995) 2463.
[36] E.A. Uehling, G. Uhlenbeck, Phys. Rev. 43 (1933) 552.
[37] M.-O. Mewes, M. Andrews, D. Kurn, D. Durfee, C. Townsend, W. Ketterle, Phys. Rev. Lett. 78 (1997) 582.
[38] I. Karatzas, Brownian Motion and Stochastic Calculus, vol. 113, Springer, 1991.
[39] A. Einstein, Investigations on the Theory of the Brownian Movement, Courier Dover Publications, 1956.
[40] R.E. Sonntag, G.J. Van Wylen, Introduction to Thermodynamics: Classical and Statistical, Wiley, New York, 1991.
[41] E. Akkalp, N. Caner, Physica A 424 (2015).
[42] E. Açıkkalp, N. Caner, Physica A 433 (2015) 247.
[43] J. Wang, J. He, Z. Wu, Phys. Rev. E 85 (2012) 031145.
[44] G. Thomas, R.S. Johal, arXiv:1410.2140, 2014.