

Interface stability in a capillary loop undergoing phase changes in non-gravitational conditions

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A mathematical model based on the Lucas-Washburn equation has been developed to address the relationships between capillary height, capillary radius and heat flux in a capillary loop. The stability criteria at the interface are studied in detail by introducing a small perturbation to the interfaces of the capillary loop. The formulae deduced as a consequence are used to analyze the influence of height of the capillary wick and the stability in a capillary loop undergoing phase changes.

capillary column, phase change, Lucas-Washburn equation, stability, non-gravitational condition

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Capillary pumped loops (CPLs) and looped heat pipes (LHPs) are structures comprising two-phase heat-transport devices that are capable of passively transporting heat over large distances with minimal temperature loss and without moving parts to pump the working fluid. For this reason, CPL and LHP structures are widely emerging as standards in designs of thermal control system for spacecrafts. Two complementary effects establish its operation. On the one hand, with the increase in heat load, the evaporating interface gradually turns inward in the capillary wick resulting in the evaporator drying out. On the other hand, the condensing interface of the liquid line fluctuates because of non-uniformity in the condensing heat load and pressure oscillations, creating temperature oscillations of the system. Therefore, stability studies of the evaporating and condensing interfaces are paramount [1]. Many researchers have demonstrated the use of phase equilibrium theory to investigate the thermodynamic behavior of the capillary interfaces [2,3], although little attention has been paid to the

effect of capillary liquid height on the capillary force and stability of the system. Moreover, the main experimental investigations were performed under gravitational conditions [4–6]. Our aim is to extend the Lucas-Washburn description to include capillary loops with phase changes and investigate the influence of capillary liquid height on the capillary force and stability of the loop by exploiting some simplifications under non-gravitational conditions.

1 Theoretical framework

Lucas [7] and Washburn [8] independently derived the force equilibrium equation for liquid rise in a capillary column based on the flow behavior of the liquid. The contributing factors include capillary force, gravity, inertia and viscosity of the liquid. As depicted diagrammatically in Figure 1, the capillary force F_c is equal to $2\pi r\sigma$, $F_g = \rho g \pi r^2 s$ is the gravitational force, $\rho \pi r^2 \frac{d(su)}{dt}$ is the inertia and the viscosity of the liquid F_v is equal to $8\pi\mu su$. F_a and F_b are the absolute

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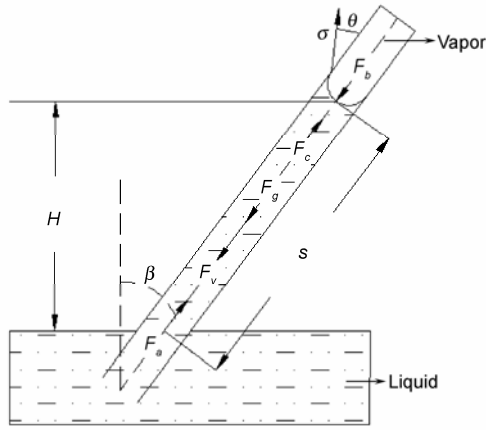


Figure 1 Force balance in the liquid column.

pressure force of the upper and bottom surface in the liquid column, respectively. According to the Lucas-Washburn equation, we have

$$F_c \cos \theta + F_a - F_b - F_g \cos \beta - F_v = \frac{d(mu)}{dt} = \frac{d(\rho \pi r^2 su)}{dt}, \quad (1)$$

where θ is the contact angle, r is the capillary radius, ρ is the liquid density, s is the axial distance of the liquid in the capillary column, β is the tilt angle of the column with the vertical direction, μ is the viscosity of the liquid, u is the average axial velocity of the liquid, and t is time. Considering the effect of vapor recoil on the behavior of the interface [9], eq. (1) can be written as

$$\frac{2\sigma \cos \theta}{r} - \rho g s \cos \beta - \Delta p - \frac{\dot{m}^2}{\rho_v} - \frac{8\mu}{r^2} su = \frac{\rho d(su)}{dt}, \quad (2)$$

where $\Delta p = \frac{F_b - F_a}{\pi r^2}$ is the pressure difference between the two ends of the liquid column, and \dot{m} is the mass flow rate. Continuity at the interface requires

$$\rho(u - \frac{ds}{dt}) = \rho_v(u_v - \frac{ds}{dt}) = \dot{m}, \quad (3)$$

where the subscript v denotes the corresponding properties of the vapor, while $\frac{ds}{dt}$ is the velocity of the interface, and thus the instantaneous velocity of the entire liquid column. According to eq. (3), we obtain

$$u = \frac{\dot{m}}{\rho} + \frac{ds}{dt}. \quad (4)$$

Substituting eq. (4) into eq. (2), we find the following eq. (5):

$$\frac{2\sigma \cos \theta}{r} - \rho g s \cos \beta - \Delta p - \frac{\dot{m}^2}{\rho_v} - \frac{8\mu \dot{m}}{r^2 \rho} s - \frac{8\mu}{r^2} s \frac{ds}{dt} = \frac{\rho}{dt} d \left(s \frac{\dot{m}}{\rho} + s \frac{ds}{dt} \right). \quad (5)$$

Here we assume that the liquid is completely wet, and under steady state conditions, eq. (5) reduces to

$$\frac{2\sigma}{r} - \Delta p - \frac{\dot{m}^2}{\rho_v} - \frac{8\mu \dot{m}}{r^2 \rho} s = 0. \quad (6)$$

Eq. (6) shows that the capillary force will balance not only the pressure difference across the interface and the vapor recoil, but also the viscosity of the liquid in the steady state.

2 Stability of the interfaces

A sketch of a capillary loop is shown in Figure 2. The dynamics of spontaneous capillary penetration into the wick is investigated by treating the media as parallel cylinders with the same radius and the flow in the wick is considered as one dimensional. For the evaporator, we denote the height of the wick by H , the capillary rise in the wick by s , and the capillary radius by r . For the condenser, the length of the capillary liquid line is $H_c = 0.3$ m, and the radius is $R = 1 \times 10^{-3}$ m.

2.1 Stability of the evaporating interface

During normal operations, the meniscus of the evaporating interface adjusts itself to balance the pressure difference Δp across the interface. To enhance the capillary force, sinter with a magnitude of 10^{-6} m in the radius provides the capillary action of the wick. As a consequence, the vapor recoil can be neglected. Under non-gravitational conditions, eq. (5) reduces to

$$\frac{2\sigma}{r_e} - \Delta p - \frac{8\mu \dot{m}}{r^2 \rho} s - \frac{8\mu}{r^2} s \frac{ds}{dt} = \frac{\rho}{dt} d \left(s \frac{\dot{m}}{\rho} + s \frac{ds}{dt} \right). \quad (7)$$

Here $\Delta p = p_v - p_l$, with p_v and p_l denoting respectively the vapor pressure at the interface and the liquid pressure at the bottom of the liquid column. In steady state, s is equal to H . We now consider the stability of the steady interfacial position H by introducing a small perturbation from equilibrium in the form

$$s = H + \varepsilon H \quad (0 < |\varepsilon| \ll 1). \quad (8)$$

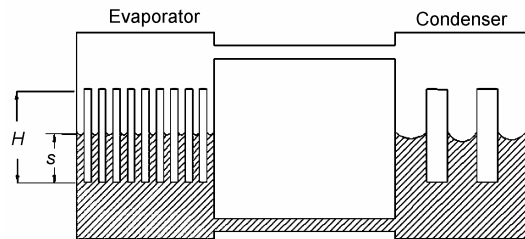


Figure 2 Sketch of a capillary loop.

Substituting eq. (8) into eq. (7), and according to Ramon’s model [10], we can obtain the following equation:

$$\varepsilon''(t) + (a + \varepsilon'(t)) \varepsilon'(t) + b\varepsilon(t) = 0, \quad (9)$$

where $a = \frac{u_0}{H} + \frac{8\mu}{\rho r^2}$, $b = \frac{u_0}{H} \frac{8\mu}{\rho r^2}$, and u_0 is the velocity of the liquid in the steady state. For eq. (9), we impose initial conditions $\varepsilon(0) = \varepsilon_0$, $\varepsilon'(0) = 0$, with $\varepsilon_0 H$ as the initial displacement from equilibrium. The working liquid is methanol, physical properties of which are shown in Table 1.

Figure 3 shows the evolution of the small displacements for two different heat fluxes. As can be seen, the time taken for the interface to return to its equilibrium position decreases with increasing heat flux, indicating that both the recovery speed of the perturbed interface to return to equilibrium and the stability of the evaporating interface are enhanced with higher heat fluxes. Actually, within the capillary wick, the interface will gradually recede into the capillary wick with the increase of the heat flux, and penetrate through the wick when the heat flux is sufficiently high, causing the wick to dry out and the system to break down. The reason for this situation to occur is probably that a large number of bubbles are created on the bottom of the wick, and the micro-channels of the flow liquid become blocked by these bubbles.

Time evolution of the small displacements for two different capillary heights is shown in Figure 4. The rate of return to the quasi-equilibrium position is slower for shorter capillary wicks. Thus, the stability can be enhanced by reducing the height of the capillary wick. However, as the liquid on the bottom of the wick will be heated quickly due to

Table 1 Physical properties of methanol at 37°C [12]

	ρ_l (kg m ⁻³)	ρ_v (kg m ⁻³)	μ (kg s ⁻¹ m ⁻¹)	σ (N m ⁻¹)	h_{fg} (J kg ⁻¹)
Methanol	7.78×10^2	0.48	4.8×10^{-4}	2.11×10^{-2}	1.14×10^6

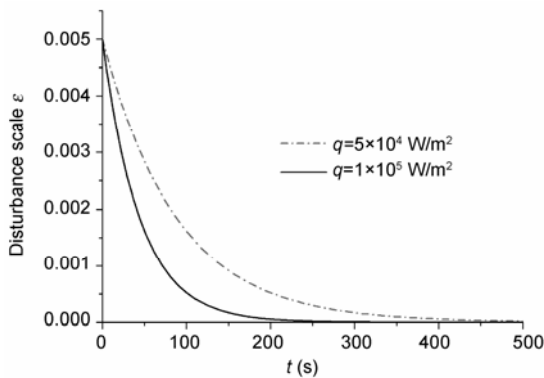


Figure 3 Evolution of small displacements at different heat fluxes ($H=5 \times 10^{-3}$ m).

high thermal conductivity, vapor may bubble up and block the flow channel of the wick. As a consequence, an appropriate capillary wick height is necessary for proper functioning.

Figure 5 shows the evolution of the small displacements for two different capillary radii. For the same relative displacement, the rate of decline for the two capillary radii is nearly the same. For this reason, a capillary structure, such as sinter nickel, with a smaller capillary radius is used in the evaporator to enhance the operational performance of the capillary loop.

Figure 6 shows the time evolution of the relative velocity $\varepsilon'(t)$. The magnitude of the velocity is found to be extremely small, taking about 3 min for the displaced system to return to its quasi-equilibrium position. As $\varepsilon'(t)$ is negligible compared with a (about 4.91 m^{-1}), eq. (9) can then be written as

$$\varepsilon''(t) + a\varepsilon'(t) + b\varepsilon(t) = 0. \quad (10)$$

Eq. (10) describes a standard over-damped system. Here, not only is the discriminant $\Delta = a^2 - 4b$ positive but also coefficients a and b , and thus the interface displays strong stability in the evaporating state.

2.2 Stability of the condensing interface

In the condensing state, we have $a = \frac{u_0}{H_c} + \frac{8\mu}{\rho R^2}$ and

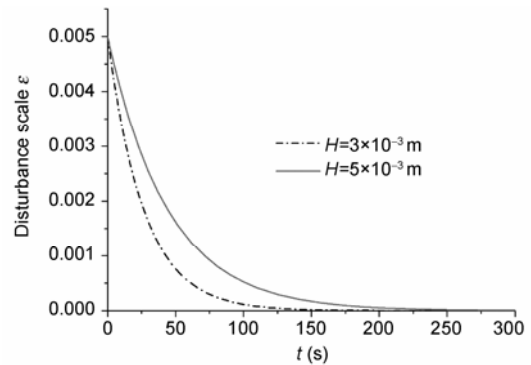


Figure 4 Evolution of the small displacements for different capillary radii ($q=1 \times 10^5 \text{ W/m}^2$).

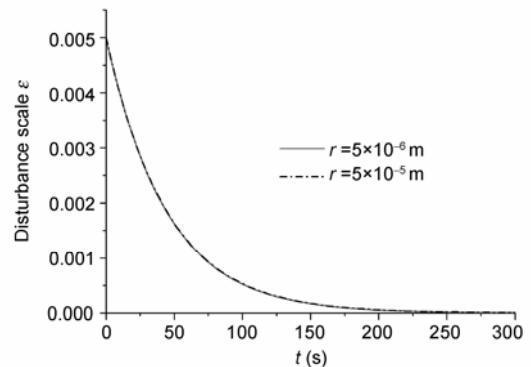


Figure 5 Evolution of the small displacements for different capillary radii ($H=5 \times 10^{-3}$ m, $q=1 \times 10^5 \text{ W/m}^2$).

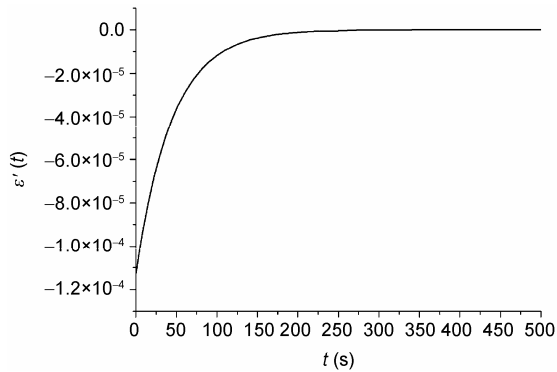


Figure 6 Evolution of the relative velocity $\varepsilon'(t)$ ($H=5 \times 10^{-3}$ m, $q=1 \times 10^5$ W/m², $r=5 \times 10^{-6}$ m).

$b = \frac{u_0}{H_c} \frac{8\mu}{\rho R^2}$ in eq. (9). For illustrative purposes, we have

set $H_c=0.3$ m and $R=1 \times 10^{-3}$ m. Also, u_0 is the steady state velocity of the liquid in the condensing pipe, and is negative because it is in the opposite sense to the evaporator capillary flow. The study of heat transfer in the condenser usually focuses on the added heat Q to the evaporator in the system.

Figure 7 shows the evolution of positive and negative small displacements under different heat loads. Both positive and negative displacements grow at the condensing interface, and these displacements increase more rapidly

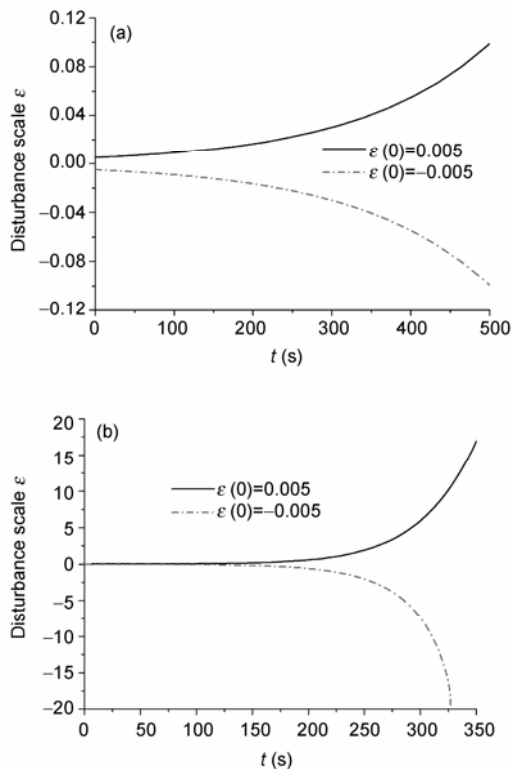


Figure 7 Evolutions of the small displacements for different heat fluxes. ($H=0.3$ m, $R=1 \times 10^{-3}$ m). (a) $Q=5$ W; (b) $Q=20$ W.

with increasing heat load. For small negative displacements, the condensing interface has a strong tendency to move towards the liquid; however, the liquid is incompressible, and thus large instabilities form at the condensing interface. For small positive displacements, the interface tends to move towards the vapor while the vapor rushes towards the interface at high speed. This tendency can be restrained by vapor recoil [10], and as a consequence, collisions in the interface result in large instabilities forming.

Figure 8 displays the effects of different condensing line lengths on the stability of the condensing interface. The instability declines with increasing length. A two-fold increase in length leads to a steep decline in the displacement by several orders of magnitude. Hence, the length of the condensing line has an obvious influence on the stability of the condensing interface, increasing the length of liquid line is helpful in enhancing the stability of condensing interface. Unfortunately, the flow resistance will also be enhanced as the liquid length increases. In this circumstance, an appropriate length for the condensing line should be found in any practical design of the system.

The time evolution of the relative velocity $\varepsilon'(t)$ of the condensing interface is shown in Figure 9. For both positive and negative small displacements, the absolute value of the relative velocity $\varepsilon'(t)$ increases with the time; hence, compared with a , $\varepsilon'(t)$ cannot be neglected in eq. (9) in the condensing state. According to the mathematical analysis of eq. (9), $\varepsilon'(t)$ can be neglected as long as $b > 0$, and in this case, the system will show strong stability. Based on the analysis above, the instability of the condensing interface involves negative values of b .

3 Conclusions

The Lucas-Washburn equation, describing the motion of a liquid body in a capillary tube has been extended so as to

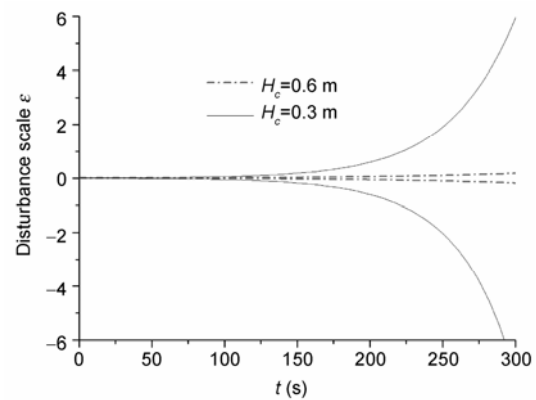


Figure 8 Evolution of the small disturbances for two different lengths of the condensing line ($Q=20$ W).

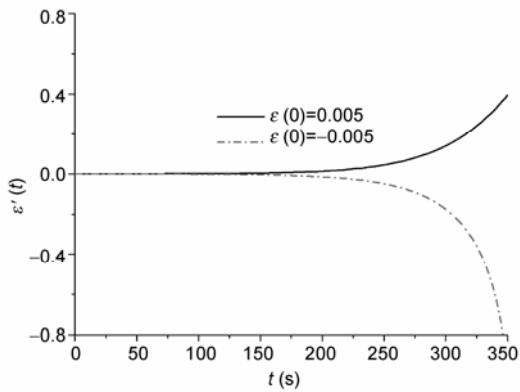


Figure 9 Evolution of the relative velocity $\varepsilon'(t)$ in the condensing state ($R=1 \times 10^{-3}$ m, $Q=20$ W).

account for the effect of height of the capillary rise on the stability of the interface allowing for phase changes—either evaporation or condensation—in a capillary loop. Based on the analysis above, we make the following conclusions:

(1) The evaporating interface is in the over-damped state. For a given small disturbance, the interface will revert rapidly to its quasi-equilibrium state, taking less time the higher the heat flux is. However, vapor may form on the bottom of the capillary wick that may block the capillary pores, causing the interface to gradually turn inwards in the wick, resulting in the system drying out.

(2) The condensing interface shows high instability. Both positive and negative small displacements are magnified in the condensing interface, and the instability enhanced with

increasing heat load and decreasing condensing line length.

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