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Coefficient of performance and its bounds with the figure of merit for a general refrigerator

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Abstract

A general refrigerator model with non-isothermal processes is studied. The coefficient of performance (COP) and its bounds at maximum χ figure of merit are obtained and analyzed. This model accounts for different heat capacities during the heat transfer processes. So, different kinds of refrigerator cycles can be considered. Under the constant heat capacity condition, the upper bound of the COP is the Curzon–Ahlborn (CA) coefficient of performance and is independent of the time durations of the heat exchanging processes. With the maximum χ criterion, in the refrigerator cycles, such as the reversed Brayton refrigerator cycle, the reversed Otto refrigerator cycle and the reversed Atkinson refrigerator cycle, where the heat capacity in the heat absorbing process is not less than that in the heat releasing process, their COPs are bounded by the CA coefficient of performance; otherwise, such as for the reversed Diesel refrigerator cycle, its COP can exceed the CA coefficient of performance. Furthermore, the general refined upper and lower bounds have been proposed.

Keywords: refrigerators, coefficient of performance, non-isothermal processes

(Some figures may appear in colour only in the online journal)

1. Introduction

In equilibrium themodynamics, Carnot's theorem states the upper bounds for energy conversion devices operating between two heat reservoirs; that is, $\eta_C = 1 - T_c/T_h$ for heat engines and $\varepsilon_C = T_c/(T_h - T_c)$ for refrigerators, where T_c and T_h are the temperatures of the cold and hot reserviors, respectively. These upper bounds have a profound impact on theoretical physics. However, in the ideal Carnot cycle, all of the processes are quasistatic and infinitely slow, leading to vanishing power output for heat engines and a zero cooling load rate for refrigerators. Therefore, for actual demand, the cycle time duration must be considered. A finite time thermodynamic analysis has provided a perspective of optimization for real refrigerators [1].

The maximum power output is often adopted as the criterion for optimizing real heat engines. Taking only into consideration the irreversibility caused by heat transfer between the heat reservoirs and the working substance during the isothermal processes, Curzon–Ahlborn [2] proposed the paradigmatic endoreversible model and deduced its efficiency at maximum power output. That is the groundbreaking CA efficiency, which is also obtained through the linear irreversible model [3]. In addition, Esposito *et al* [4] proposed the low dissipation model by considering the entropy generation in the isothermal processes, which are treated as an inversed function of process time duration in the heat absorbing and releasing processes, respectively, and then they obtained the lower and upper bounds of the efficiency at maximum power output. Furthermore, the CA efficiency is recovered under the symmetric dissipation condition. Considering the temperature changes in the heat absorbing and releasing processes and that the heat transfer obeys Newton's law of cooling, Yan and Guo [5] also obtained the upper and lower bounds of efficiency at maximum power output. In addition, the same bounds are also proposed by Izumida and Okuda through the minimally nonlinear irreversible model [6].

However, for refrigerators, the minimum power input is not an appropriate optimization criterion [7], and much effort has been devoted to optimizing refrigerators under different figures of merit. Jiménez de Cisneros *et al* [8] studied the COP at maximum COP criterion through the linear irreversible model. By maximizing the per-unit-time COP, Velasco *et al* [9] obtained the upper bound of COP, $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, i.e. the CA coefficient of performance, for endorevesible refrigerators. It has been also obtained in refrigerators with non-isothermal processes [10]. Hernandez *et al* [11] proposed the Ω figure of merit, indicating a compromise between energy benefits and losses. In subsequent studies, some efforts have been shown to be delicate to the optimization using the Ω criterion. De Tomas *et al* [12] and Hernandez *et al* [13] investigated the COP of the refrigerators based on the Ω figure of merit and proposed the upper and lower bounds of the COP. The same bounds are also obtained through the minimally nonlinear irreversible model under the tight-coupling conditions [14]. Furthermore, Yan and Chen [15] conducted the optimization with the target function $\varepsilon \dot{Q}_c$, where $\dot{Q}_c$ is the cooling load rate of the refrigerators. De Tomas *et al* [16] introduced the unified optimization criterion χ both for heat engines and refrigerators. This criterion is defined as the efficiency or COP multiplied by the heat absorbed Q_{in} divided by the cycle time duration t_{cycle}, i.e. $\chi = zQ_{\text{in}}/t_{\text{cycle}}$, where z is the efficiency for heat engines or the COP for refrigerators. By taking χ as the target function, based on the low dissipation model, Wang *et al* [7] proposed that the COP at maximum χ was bounded between 0 and $\left(\sqrt{9 + 8\varepsilon_C} - 3\right)/2$. In addition, the same bounds are also obtained through the minimally nonlinear irreversible model [17].

The main merit of the low dissipation models and the linear irreversible and minimally nonlinear irreversible models is that we do not need to consider the heat transfer law between the working medium and the heat reservoirs. However, in the low dissipation model, the temperature of the working medium does not change during heat transferring processes, which is not true for a realistic system. In the linear irreversible model, the temperature difference of the cold and hot reservoirs should be small enough to meet the requirement of the Onsager relations. In reality, the heat exchanging processes should not be isothermal, and some attention has been focused on considering the temperature changes of the working substance during the heat absorbing and releasing processes [18]. In addition, the tradional ones, which specify concrete heat transfer laws, are still of merit. Based on the heat transfer law, the connection between the maximum-work and maximum-power thermal cycles has also been discussed [19]. In this paper, we use the method proposed in [5] to describe the refrigerator cycle. This model accounts for the temperature changes of the working substance in heat exchanging processes. Therefore, it is more general and realistic. First, we introduce the model in section 2. Then, the COP of refrigerators and its bounds are systematically analyzed based on the χ figure of merit in sections 3 and 4. The new COP bounds are proposed. Finally, some concluding remarks are given.

2. Mathematical model

As to refrigerators, the cooling load Q_c is absorbed from the cold reservoir (T_c), and a certain amount of heat Q_h is evacuated to the hot reservoir (T_h) at the end of a cycle. The heat transfer law between the heat source and the working substance is assumed to conform to Newton's law of cooling

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = cm\frac{\mathrm{d}T}{\mathrm{d}t} = k\left(T_s - T\right) \tag{1}$$

where c is the heat capacity, m is the working substance mass, T is the working substance temperature, T_s is the heat source temperature and k is the heat conductance (contact area multiplied by the heat transfer coefficient). The initial temperature of the working substance is T_{c0} and T_{h0} at the beginning of the cooling and heating processes, respectively. According to equation (1), the working substance temperature in the heat absorbing process is a function of time t

$$T = T_c - \left(T_c - T_{c0}\right)\mathrm{e}^{-t/\sum_c} \tag{2}$$

where $\sum_c = c_c m/k_c$. The time duration of the heat absorbing process is τ_c, and the cooling load can be calculated as

$$Q_c = \int k_c\left(T_c - T\right) = c_c m\left(T_c - T_{c0}\right)\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right) \tag{3}$$

The relative entropy change of the working substance in the heat absorbing process is given by

$$\Delta s_c = \int \frac{dQ_c}{T} = c_c m \ln \frac{T_c - \left(T_c - T_{c0}\right)\mathrm{e}^{-\tau_c/\sum_c}}{T_{c0}} \tag{4}$$

Similarly, the heat evacuated to the hot reservoir and the entropy change during the heat releasing process are given by

$$Q_h = c_h m\left(T_{h0} - T_h\right)\left(1 - \mathrm{e}^{-\tau_h/\sum_h}\right) \tag{5}$$

$$\Delta s_h = -c_h m \ln \frac{T_h + \left(T_{h0} - T_h\right)\mathrm{e}^{-\tau_h/\sum_h}}{T_{h0}} \tag{6}$$

where $\sum_h = c_h m/k_h$, τ_h is the time duration of the heat releasing process. In this paper, we assume that the compressing and expanding processes are isentropic and that the time for completing those processes is zero. After a cycle, the working substance returns to its initial state, and the total entropy change of the working substance should be zero, i.e. $\Delta s_h + \Delta s_c = 0$. Then, we have

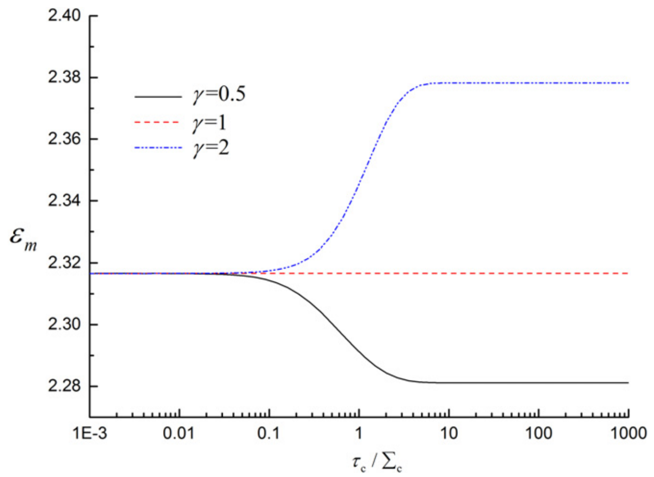

Figure 1. Optimal COP (ε_m) with dimensionless contact times of the heat absorbing process under different heat capacity ratios ($\gamma = 0.5,\ 1,\ 2$), where $\varepsilon_C = 10$ and $\tau_h/\sum_h = 1$.

$$\frac{T_c - \left(T_c - T_{c0}\right)\mathrm{e}^{-\tau_c/\Sigma_c}}{T_{c0}}\left(\frac{T_h + \left(T_{h0} - T_h\right)\mathrm{e}^{-\tau_h/\Sigma_h}}{T_{h0}}\right)^{\gamma} = 1 \quad (7)$$

where $\gamma = c_h/c_c$. The coefficient of performance ε is

$$\varepsilon = \frac{Q_c}{Q_h - Q_c} \quad (8)$$

The figure of merit $\chi = \varepsilon Q_c/t_{\mathrm{cycle}}$ is

$$\chi = \left(\left[c_c m\left(T_c - T_{c0}\right)\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\right]^2\right)\Big/ \\ \times\left(\left[c_h m\left(T_{h0} - T_h\right)\left(1 - \mathrm{e}^{-\tau_h/\Sigma_h}\right) \\ - c_c m\left(T_c - T_{c0}\right)\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\right]\left(\tau_h + \tau_c\right)\right) \quad (9)$$

Combining equations (7) and (9) and maximizing χ with respect to T_{c0}, we have

$$\frac{\varphi(\varphi - 1)}{2}\frac{\varepsilon_C + 1}{\varepsilon_C}\left(\left(1 - \mathrm{e}^{-\tau_h/\Sigma_h}\right)^2\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right) \\ \times\left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma - 1}\right)\Big/ \\ \times\left(\left\{\left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma} - \mathrm{e}^{-\tau_h/\Sigma_h}\right\}^2\right) \\ - \gamma\frac{\varepsilon_C + 1}{\varepsilon_C}\frac{1 - \left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma}}{\left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma} - \mathrm{e}^{-\tau_h/\Sigma_h}} \\ \times\left(1 - \mathrm{e}^{-\tau_h/\Sigma_h}\right) + \frac{1}{2}\left(1 - \frac{1}{\varphi}\right)\left(1 - e^{-\tau_c/\Sigma_c}\right) = 0 \quad (10)$$

where $\varphi = T_c/T_{c0}$. In the heat absorbing process, the initial temperature of the working medium should be lower than that of the cold reservoir; therefore, $\varphi > 1$. Meanwhile, substituting equations (3) and (5) for equation (8), the COP of the refrigerator can be rewritten as

$$\varepsilon = \left((\varphi - 1)\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\right)\Big/\left(\gamma\frac{\varepsilon_C + 1}{\varepsilon_C}\varphi \\ \times\frac{1 - \left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma}}{\left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]^{-1/\gamma} - \mathrm{e}^{-\tau_h/\Sigma_h}} \\ \times\left(1 - \mathrm{e}^{-\tau_h/\Sigma_h}\right) - (\varphi - 1)\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\right) \quad (11)$$

In general, the COP (ε_m) at maximum χ figure of merit can be derived using equations (10) and (11), which will be discussed in the following parts.

3. Constant heat capacity ($\gamma = 1$)

In this situation, the heat capacity stays constant during the refrigerator cycle. Thus, $\gamma = 1$. Therefore, equations (10) and (11) can be rewritten as

$$\mathrm{N}^2 - 2\mathrm{N} + \frac{\varepsilon_C}{\varepsilon_C + 1} = 0 \quad (12)$$

and

$$\varepsilon = \frac{2}{\frac{\varepsilon_C + 1}{\varepsilon_C}\mathrm{N}^2 - 1} \quad (13)$$

where

$$\mathrm{N} = \frac{\left(1 - \mathrm{e}^{-\tau_h/\Sigma_h}\right)\varphi}{1 - \mathrm{e}^{-\tau_h/\Sigma_h}\left[\left(1 - \mathrm{e}^{-\tau_c/\Sigma_c}\right)\varphi + \mathrm{e}^{-\tau_c/\Sigma_c}\right]} \quad (14)$$

Combining equations (12) and (13), we have

$$\varepsilon_m = \sqrt{\varepsilon_C + 1} - 1 \equiv \varepsilon_{CA} \quad (15)$$

Equation (15) gives the upper bound of the COP at the maximum χ criterion, which is independent of the time duration in each process and is equal to the CA coefficient of performance. It has also been obtained by using the endoreversible refrigerator model [15]. Although the upper bounds of the COP are the same for these two models, they have different physical meanings, and the optimization spaces are different. In the endoreversible model, the upper bound of the COP is obtained by maximizing χ with respect to the time durations of the heat absorbing and releasing processes, respectively, while in this model, the upper bound is obtained by maximizing χ with respect to the initial temperature of the working medium, and the time durations are treated as constants. Unlike the Carnot refrigerators, in this model, the temperature of the working medium in either heat exchanging process does not stay constant. The model studied in this paper should be more practical and realistic than the endoreversible refrigerator one. In addition, situations with various heat capacities can be considered futher in this model.

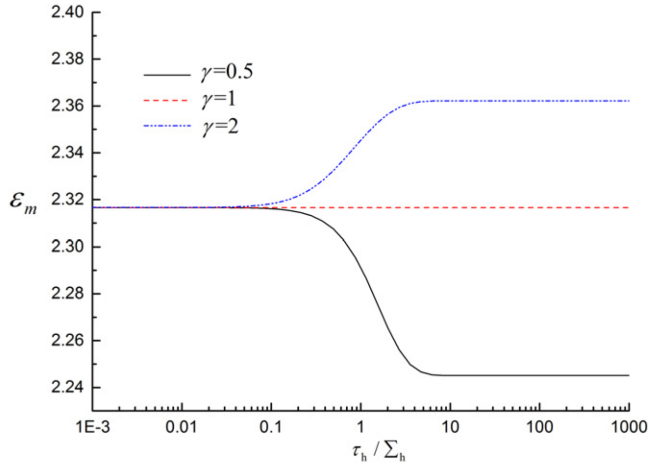

Figure 2. Optimal COP (ε_m) with dimensionless contact times of the heat releasing process under different heat capacity ratios ($\gamma = 0.5,\ 1,\ 2$), where $\varepsilon_C = 10$ and $\tau_c/\sum_c = 1$.

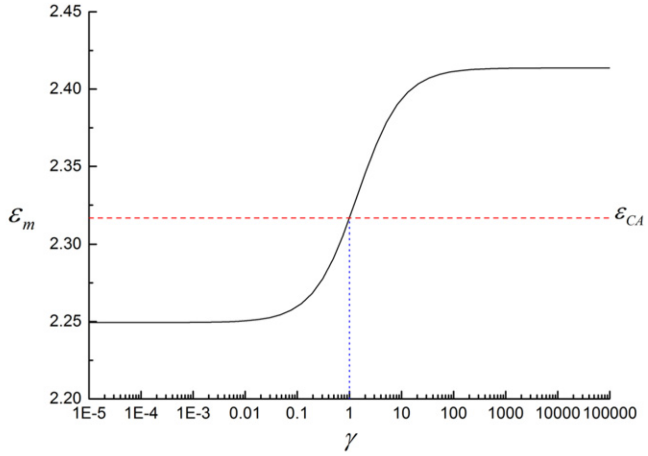

Figure 3. Optimal COP (ε_m) with different heat capacity ratios, where $\varepsilon_C = 10$ and $\tau_h/\sum_h = \tau_c/\sum_c = 1$.

4. Non-constant heat capacity ($\gamma \neq 1$)

When the heat capacities do not remain constant in the heat exchanging processes, equation (10) is transcendental and cannot be solved explicitly. Numerical calculations are conducted to investigate the impacts of the parameters on the optimal COPs. As shown in figure 1, when the heat capacity ratio is larger than unit ($\gamma > 1$), and $\tau_h/\sum_h$ is fixed, the optimal COP increases with increasing $\tau_c/\sum_c$ in a certain interval and achieves its lower and upper bound under the asymmetric limits :$\tau_c/\sum_c \to 0$ and $\tau_c/\sum_c \to \infty$, respectively. The lower bound is the CA coefficient of performance. When the heat capacity ratio is less than unit ($\gamma < 1$), the optimal COP decreases with increasing $\tau_c/\sum_c$ in a certain interval and achieves its lower and upper bound under the asymmetric limits $\tau_c/\sum_c \to \infty$ and $\tau_c/\sum_c \to 0$. The upper bound is the CA coefficient of performance. As mentioned before when the heat capacity ratio is a unit, the optimal COP is the CA coefficient of performance and is independent of $\tau_c/\sum_c$.

As we can see in figure 2, when the heat capacity ratio is larger than unit ($\gamma > 1$), and $\tau_c/\sum_c$ is fixed, the optimal COP increases with increasing $\tau_h/\sum_h$ in a certain interval and achieves its lower and upper bound when $\tau_h/\sum_h \to 0$ and $\tau_h/\sum_h \to \infty$, respectively. The lower bound is the CA coefficient of performance. When the heat capacity ratio is less than unit ($\gamma < 1$), the optimal COP decreases with increasing $\tau_h/\sum_h$ in a certain interval and achieves its lower and upper bound when $\tau_h/\sum_h \to \infty$ and $\tau_h/\sum_h \to 0$. The upper bound is the CA coefficient of performance. As mentioned before when the heat capacity ratio is a unit, the optimal COP stays constant and is the CA coefficient of performance.

To take this a step further, when $\gamma > 1$, the optimal COP will increase with increasing $\tau_h/\sum_h$ and $\tau_c/\sum_c$ and will achieve its maximum value when $\tau_h/\sum_h \to \infty$ and $\tau_c/\sum_c \to \infty$. The lower bound is achieved when $\tau_h/\sum_h \to 0$ and $\tau_c/\sum_c \to 0$ are equal to the CA coefficient of performance and are independent of the heat capacity ratio. When $\gamma < 1$, the optimal COP will decrease with increasing $\tau_h/\sum_h$ and $\tau_c/\sum_c$ and will obtain its minimum value when $\tau_h/\sum_h \to \infty$ and $\tau_c/\sum_c \to \infty$. The upper bound is achieved when $\tau_h/\sum_h \to 0$ and $\tau_c/\sum_c \to 0$ are equal to the CA coefficient of performance and are also independent of the heat capacity ratio. Therefore, in the refrigerator cycles, such as the reversed Brayton refrigerator cycle ($c_c = c_h = c_p$), the reversed Otto refrigerator cycle ($c_c = c_h = c_v$) and the reversed Atkinson refrigerator cycle ($c_h = c_v, c_c = c_p$), where the heat capacity in the heat absorbing process is not less than that in the heat releasing process, their COPs under the maximum χ criterion are bound by the CA coefficient of performance, while in the refrigerator cycles, such as the reversed Diesel refrigerator cycle ($c_h = c_p,\ c_c = c_v$), where the heat capacity in the heat absorbing process is less than that in the heat releasing process, their COPs under the maximum χ criterion can exceed the CA coefficient of performance. This might be of great guidance for selecting an appropriate working substance for designing or operating a refrigerator. Furthermore, as depicted in figure 3, when the dimensionless contact times are fixed, the optimal COP increases with the increasing heat capacity ratios in a certain interval and achieves its lower and upper bound when $\gamma \to 0$ and $\gamma \to \infty$. The CA coefficient of performance is also retrieved when $\gamma = 1$. Therefore, in the situations where $\tau_h/\sum_h \to \infty$ and $\tau_c/\sum_c \to \infty$, the general upper and lower bounds of the optimal COP can be obtained by applying the asymmetric heat capacity limits $\gamma \to \infty$ and $\gamma \to 0$, respectively. The above analysis will be studied further in the following sections.

4.1. Short contact time limits

Under the conditions where $t/\sum \to 0$, the heat absorbing and releasing processes are both so short that the final temperature of the working substance is almost equal to its initial temperature after either process. When we expand $\exp(-t/\sum)$ to the first order of $t/\sum$, equations (10) and (11) can be reduced as

$$\gamma^2 \frac{\varepsilon_C + 1}{\varepsilon_C} M^2 - 2\gamma \frac{\varepsilon_C + 1}{\varepsilon_C} M + 1 = 0 \tag{16}$$

and

$$\varepsilon = \frac{1}{\gamma \frac{\varepsilon_C + 1}{\varepsilon_C} M - 1} \tag{17}$$

where

$$M = \frac{\frac{\tau_h}{\sum_h} \varphi}{\gamma \frac{\tau_h}{\sum_h} - (\varphi - 1) \frac{\tau_c}{\sum_c}} \tag{18}$$

The solution to equation (16) gives the optimal M

$$M_{opt} = \frac{1}{\gamma} \frac{\varepsilon_C + 1 + \sqrt{\varepsilon_C + 1}}{\varepsilon_C + 1} \tag{19}$$

Substituting equation (19) into equation (17), we have the same upper bound of the COP as equation (15). It is the CA coefficient of performance and is independent of the heat capacity ratio.

4.2. Long contact time limits

As mentioned above, in the situations where $t/\sum \to \infty$, their upper and lower bounds of the optimal COP can be obtained by applying the asymmetric heat capacity limits $\gamma \to \infty$ and $\gamma \to 0$. Under the conditions where $t/\sum \to \infty$, the contact time is long enough so that the heat exchange between the working substance and heat reservoirs is sufficient, and the final temperature of the working substance is almost equal to that of the heat reservoir. The exponential terms $\exp(-t/\sum)$ can be eliminated; therefore, equations (10) and (11) are simplified as

$$(\varphi - 1)\varphi^{1/\gamma} - 2\gamma\left(\varphi^{1/\gamma} - 1\right) + \frac{\varepsilon_C}{\varepsilon_C + 1}\left(1 - \frac{1}{\varphi}\right) = 0 \tag{20}$$

and

$$\varepsilon_m = \frac{2}{\frac{\varepsilon_C + 1}{\varepsilon_C} \varphi^{1/\gamma + 1} - 1} \tag{21}$$

For symmetric capacity ratio ($\gamma = 1$), according to equation (20), we have

$$\varphi_{opt} = 1 + \sqrt{\frac{1}{\varepsilon_C + 1}} \tag{22}$$

Substituting equation (22) into equation (21), the CA coefficient of performance is also recovered. While under asymmetric conditions ($\gamma \neq 1$), equation (20) cannot be solved analytically. Based on equations (20) and (21), numerical calculations are conducted to obtain the optimal COPs, and the results are plotted in figure 4. The curves for $\gamma = 0.0001$ and $\gamma = 0.01$ coincide with each other, as do the curves for $\gamma = 100$ and $\gamma = 10000$. Therefore, the curves for $\gamma = 0.0001$ and $\gamma = 10000$ can be treated as the lower and upper bounds of the COP for the refrigerator, which are fitted as $\varepsilon_m^{-} = \left(\sqrt{9 + 8\varepsilon_C} - 3\right)/3.42$ and $\varepsilon_m^{+} = \left(\sqrt{9 + 8\varepsilon_C} - 3\right)/2.42$, respectively. The upper bound of the COP under the χ criterion through the low dissipation model is $\left(\sqrt{9 + 8\varepsilon_C} - 3\right)/2$ [7], which shares the same form with the lower and upper bounds obtained in the present paper. Furthermore, in the previous studies under the maximum χ criterion, the lower bound is zero [7, 13, 17]; however, it is far below all of the observed COPs [7, 13]. Here, we have obtained a new lower bound, which agrees well with some experimental COPs [13]. Therefore, the lower bound proposed in this paper should be more practical and realistic than the previous ones.

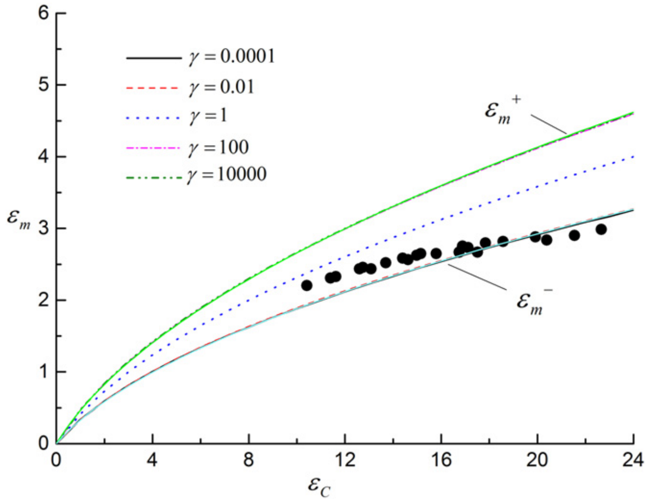

Figure 4. Optimal COP (ε_m) with the Carnot COP under different heat capacity ratios. The fitted lower (color cyan) bound and upper bound (color green) of the COP are plotted. In addition, we also plot some experimental COPs (dots) [13] in this figure.

5. Conclusions

A general refrigerator model with non-isothermal processes is studied. The coefficient of performance at maximum χ figure of merit has been analyzed systematically, and the new COP bounds have been obtained. For symmetric heat capacity ratio ($\gamma = 1$), the upper bound of the COP is ε_{CA} and is independent of the time durations of the heat exchanging processes. The same bound has been also obtained using the endoreversible refrigerator model. However, they have different physical meanings, and the optimization spaces are different. In the refrigerator cycles, such as the reversed Brayton refrigerator cycle, the reversed Otto refrigerator cycle and the reversed Atkinson refrigerator cycle, where the heat capacity in the heat absorbing process is not less than that in the heat releasing process, their COPs under the maximum χ criterion are bounded by the CA coefficient of performance, while in the refrigerator cycles, such as the reversed Diesel refrigerator cycle, where the heat capacity in the heat absorbing process is less than that in the heat releasing process, their COPs under the maximum χ criterion can exceed the CA coefficient of performance. Furthermore, the COP under the maximum χ criterion in two special cases have been studied, and the general upper and lower bounds have been proposed.

However, the present model does not take into consideration the internal irreversibility of the refrigerator cycles. In real-life refrigerator cycles, the compressing and expanding processes are no longer isentropic, and many models have been proposed to investigate the irreversibility in those processes [20, 21]. Furthermore, impacts of frictional losses on the COP are also studied for traditional and quantum refrigerators [22, 23]. Extensions of the present model concerning the internal irreversibility need to be studied further.

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