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Convective heat transfer optimization based on minimum entransy dissipation in the circular tube

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ABSTRACT

Convective heat transfer optimization based on minimum entransy dissipation is studied in this paper. By setting entransy dissipation as optimization objective and power consumption as constraint condition, optimized fluid momentum equation with additional volume force for convective heat transfer are deduced by variational principle. Numerical investigations for convective heat transfer in a straight circular tube based on optimized governing equations are conducted. The results show that there exist longitudinal swirl flows with multi-vortexes in the tube, which leads to heat transfer enhancement at relatively small flow resistance. The present analysis for heat transfer and flow shows that this kind of optimized flow field can realize a far greater increase in heat transfer than that in flow resistance, which indicates that the investigated optimization method is useful in design of heat transfer enhancement.

1. Introduction

As far as heat transfer enhancement [1,2] is concerned, many researchers focus on whether heat transfer is enhanced and to which degree it is enhanced, while ignoring the increase in flow resistance which sometimes may exceed the degree of heat transfer enhancement. With growing concern about energy saving in heat exchangers which are widely used in industry, more and more researchers are devoted to developing heat transfer enhancement unit which can work efficiently with low power consumption. Since the overall performance of convective heat transfer is heavily dependent on heat transfer process, great emphasis should be laid upon the process optimization [3]. However, the currently used methods in heat transfer enhancement are more technical and lack of theoretical optimization to guide the design for various enhancement techniques.

Bejan et al. [4–7] proposed the constructal theory, which simplified the complicated geometric construction into the assembly of a series of fundamental units, and made the transport process optimization possible. Guo et al. [8] proposed the field synergy principle in analyzing the relationship between the local behavior and the overall performance of convective heat transfer in two-dimensional laminar flow. They pointed out that the performance of convective heat transfer was dependent on the synergy between temperature and velocity fields. With the same velocity and temperature boundary conditions, the larger the synergy degree was, the better the convective heat transfer would be. Based on the field synergy principle, Liu et al. [9–11] considered multi-field synergy in convective heat transfer by reexamining the physical mechanism of convective heat transfer between fluid and solid wall in the laminar and turbulence flows. They revealed how heat transfer enhancement was influenced by multi-field synergy relation associated with temperature, velocity and pressure and explained physical essentials on enhancing heat transfer and reducing flow resistance. According to the field synergy principle, we can know that the performance of convective heat transfer is dependent on the organization of fluid field, and what we need to do is to find an optimized fluid field. After finding it, a heat transfer enhancement solution which is closest to this optimized fluid field can be identified and implemented.

Bejan deduced entropy generation expression and analyzed optimization parameters in heat exchangers or heat transfer systems by taking minimum entropy generation as optimization objective [12,13] induced by heat transfer and viscous dissipation. Xia [3] set thermal potential loss as the evaluation objective and used viscous dissipation to denote the loss in mechanical energy. With fixed mechanical energy loss, the optimized velocity field equation can be derived through functional analysis, in which a scalar item was unknown. Meng [14] furthered Xia's analysis by using Lagrange multiplier to make functional analysis. The field synergy equation was derived, and each term in the equation

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Nomenclature

A, B, C_0	Lagrange multipliers
c_p	specific heat at constant pressure, J/(kg K)
e_h, ϕ_h	entransy dissipation, W K/m^3
E_{vh}	entransy, J K
$\boldsymbol{F}$	volume force vector, N
J	functional
p	pressure, Pa
Q_{vh}	heat capacity, J
T	temperature, K
$\boldsymbol{U}$	velocity vector, m/s
V	volume, m^3
W_p	power consumption, W
Greek symbols	
λ	thermal conductivity, W/(m K)
ρ	fluid density, kg/m^3
μ	viscosity coefficient, kg/(m s)
Ω	control volume, m^3
Γ	control surface, m^2

was identified. Based on his analysis, the heat-transfer enhanced tubes with two longitudinal vortexes were designed, which exhibited better heat transfer performance. Guo et al. [15] proposed a new physical quantity – entransy to describe the capability of heat energy transportation. They also proposed the principle of entransy dissipation extremum [16–21] to optimize heat transfer process by setting fixed viscous dissipation as constraint condition. Compared with the principle of minimum entropy generation, the principle of entransy dissipation extremum is more suitable in heat transfer process optimization. Chen et al. [22–27] optimized heat exchanger, constructal problems of variable cross-section channel, "volume-point" heat conduction, and so on, with minimum entransy dissipation rate as optimization objective.

The power consumed in incompressible fluid flow is partly stemmed from the fluid viscosity reflecting frictional resistance and profile resistance, and partly from the momentum change. In order to reach a maximum amount of heat transfer without excessive power consumption, we can set minimum entransy dissipation as optimization objective and fixed power consumption as constraint condition in developing optimization method.

2. Convective heat transfer optimization

Based on the analogies between thermal and electrical conductions, Guo defined the entransy as half of the product of heat capacity and temperature:

$$E_{vh} = \frac{1}{2} Q_{vh} T \tag{1}$$

where T is temperature, Q_{vh} is heat capacity at constant volume in general. The entransy dissipation function which represents entransy dissipation per unit time and per unit volume was deduced as [15]:

$$\phi_h = \lambda (\nabla T)^2 \tag{2}$$

where λ is thermal conductivity, and ∇T is temperature gradient.

By setting entransy dissipation as optimization objective and viscous dissipation as constraint condition, optimization flow field equation for convective heat transfer was constructed by Meng [14] as:

$$\rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} + \nabla p - \mu \nabla^2 \boldsymbol{U} - \left(\frac{\rho c_p}{2C_0} A \nabla T + \rho \boldsymbol{U} \cdot \nabla \boldsymbol{U} \right) = 0 \tag{3}$$

where μ is viscosity coefficient, $\boldsymbol{U}$ is velocity vector, ρ is fluid density, p is pressure, c_p is specific heat at constant pressure, C_0 is constant Lagrange multiplier. Scalar Lagrange multiplier A satisfies the following equation:

$$\rho c_p \boldsymbol{U} \cdot \nabla A + \lambda \nabla^2 A - \lambda \nabla^2 T = 0 \tag{4}$$

3. Optimized field equations

As we know that heat transfer enhancement is usually accompanied by an undesirable increase in flow resistance. So when dealing with problems of heat transfer enhancement, we need to take both thermal and flow resistances into consideration. As it is mentioned from above, entransy dissipation which can also be defined as entransy dissipation to represent the irreversibility of a heat transfer process, is a physical quantity to measure the loss of heat transfer capability, so we can use it to evaluate the intensity of convective heat transfer enhancement, which is expressed as [15]:

$$e_h = \lambda (\nabla T)^2 \tag{5}$$

The entransy dissipation can be regarded as an expression of the irreversibility of heat transfer process, which is similar as the entropy generation to that of thermodynamics process. Entransy always decreases in heat transfer process, while entropy always increases. The smaller the entransy dissipation, the smaller the temperature difference of the fluid is, and thereby the smaller the irreversibility of the heat transfer process is [28].

In the flow of incompressible fluid, the power consumption, partly from the fluid viscosity and partly from the momentum change, can be expressed as:

$$W_p = -\boldsymbol{U} \cdot \nabla p = \boldsymbol{U} \cdot [\rho (\boldsymbol{U} \cdot \nabla) \boldsymbol{U} - \mu \nabla^2 \boldsymbol{U}] \tag{6}$$

From Eq. (6), it can be seen that the power consumption is correlated with the velocity field, meanwhile heat transfer characteristics and thermal resistance of the fluid are correlated with the temperature field coupling with velocity distribution. Therefore, optimizing a convective heat transfer process is to find an optimal velocity field with fixed power consumption which satisfies minimum entransy dissipation leading to small thermal resistance. This is a typical problem of functional variational. Its deduction is described below.

(1) Optimization target: fluid velocity field
(2) Optimization objective: entransy dissipation extremum expressed as variation:

$$\delta \int_\Omega e_h dV = 0 \tag{7}$$

(3) Constraint conditions:

Fixed power consumption:

$$\int_\Omega W_p dV = const \tag{8}$$

Mass conservation of incompressible fluid:

$$\nabla \cdot \boldsymbol{U} = 0 \tag{9}$$

Energy conservation ignoring viscous dissipation:

$$\lambda\nabla^2 T - \rho c_p \boldsymbol{U}\cdot\nabla T = 0 \tag{10}$$

(4) Boundary conditions:

Constant velocity at boundaries, expressed as variation:

$$\delta \boldsymbol{U}|_\Gamma = 0 \tag{11}$$

Constant viscous shearing stress at boundaries [29], expressed as variation:

$$\delta(\mu\nabla\boldsymbol{U})|_\Gamma = \delta(\nabla\boldsymbol{U})|_\Gamma = 0 \tag{12}$$

Constant wall temperature or heat flux boundaries, expressed as variation:

$$\delta T|_\Gamma = 0 \text{ or } \delta(\lambda\nabla T)|_\Gamma = \delta(\nabla T)|_\Gamma = 0 \tag{13}$$

A function can be constructed by Lagrange multipliers as:

$$J = \int_\Omega \{\lambda(\nabla T)^2 + C_0\boldsymbol{U}\cdot[\rho(\boldsymbol{U}\cdot\nabla)\boldsymbol{U} - \mu\nabla^2\boldsymbol{U}] + A\nabla\cdot\boldsymbol{U} + B(\lambda\nabla^2 T - \rho c_p\boldsymbol{U}\cdot\nabla T)\}dV \tag{14}$$

where multiplier C_0 is constant, multipliers A and B are unknown scalar functions. After finding functional variation of Eq. (14) with respect to velocity $\boldsymbol{U}$ and temperature T respectively, we can have the following equations. The variation of Eq. (14) with respect to T and $\boldsymbol{U}$ yields the following equations [30]:

Within the region Ω:

$$\rho[\boldsymbol{U}\cdot\nabla\boldsymbol{U} + \boldsymbol{U}\times(\nabla\times\boldsymbol{U})] - 2\mu\nabla^2\boldsymbol{U} - \frac{\nabla A}{C_0} - \frac{\rho c_p B\nabla T}{C_0} = 0 \tag{15}$$

$$-2\lambda\nabla^2 T + \rho c_p\boldsymbol{U}\cdot\nabla B + \lambda\nabla^2 B = 0 \tag{16}$$

On the boundary Γ:

$$(2\lambda\nabla T - \rho c_p\boldsymbol{U}B - \lambda\nabla B)\delta T + \lambda B\delta(\nabla T) = 0 \tag{17}$$

$$(\rho C_0\boldsymbol{U}\cdot\boldsymbol{U} + C_0\mu\nabla\boldsymbol{U})\cdot\delta\boldsymbol{U} + A\delta\boldsymbol{U} - C_0\mu\boldsymbol{U}\cdot\delta(\nabla\boldsymbol{U}) = 0 \tag{18}$$

For the momentum conservation, we have:

$$\rho(\boldsymbol{U}\cdot\nabla)\boldsymbol{U}] = -\nabla p + \mu\nabla^2\boldsymbol{U} + F \tag{19}$$

By making an analogy [31] between Eqs. (16) and (19), we let

$$\rho\boldsymbol{U}\times(\nabla\times\boldsymbol{U}) = \frac{\nabla A}{C_0} + \nabla p + \mu\nabla^2\boldsymbol{U} \tag{20}$$

where unknown scalar A is determined by Eq. (20) and boundary condition (18).

Then Eq. (15) can be written as:

$$\rho\boldsymbol{U}\cdot\nabla\boldsymbol{U} = -\nabla p + \mu\nabla^2\boldsymbol{U} + \frac{\rho c_p B}{C_0}\nabla T \tag{21}$$

where unknown scalar B is determined by Eq. (16) and boundary condition (17).

Eq. (21) is the final optimized momentum equation, and its last term on the right hand represents the additional virtual volume force to organize optimal flow field, which keeps entransy dissipation minimum when power consumption is fixed.

4. Optimized field analysis for convective heat transfer in laminar flow

Heat transfer enhancement of laminar flow inside straight circular tube which is a commonly used heat-transfer unit in the tube-and-shell heat exchangers, which is widely studied in engineering. In order to verify the effect of optimization field equations deduced above, a computational model shown in Fig. 1 is introduced. Tube inner diameter D is 20 mm, and tube length is 1700 mm. The tube is divided into three sections: entrance length L_1, optimization length L_2, exit length L_3, and their sizes are 1200, 300 and 200 mm respectively. The fluid flowing inside the tube is water. Inlet water temperature is set as 300 K, and tube wall temperature 310 K.

The governing equations in the numerical computation include continuity equation, energy equation, optimized momentum equation and scalar B equation. The governing equation and the boundary condition for unknown scalar B is determined by Eqs. (15) and (19), respectively. The CFD software Fluent 6.3 is used for solving the coupled governing equations, and the SIMPLEC algorithm is used for coupling pressure and velocity fields. The QUICK discrete scheme is applied in the momentum and energy equations. To solve the constraint scalars, we use the user defined function (UDF) in the Fluent.

In the calculation, we tried different values of Re number and W_p which is the power consumption within the optimization section of 300 mm. The temperature and velocity fields of cross-section of optimization section at the position z = 1350 mm were observed and analyzed. Fig. 2 shows the temperature and flow fields in the cross-section of tube at above mentioned position when Re = 200 and $W_p = 8.01\times10^{-7}$ W, the intensity of secondary flow is 0.034% of section average flow velocity. Fig. 3 is for the case when Re = 200 and $W_p = 8.36\times10^{-7}$ W, the intensity of secondary flow is 0.073% of section average flow velocity. The observed results show that for the same Re number, the solution varies with W_p within a certain range. If W_p is beyond this range, too small or too large, it would be impossible to obtain convergence solution. When W_p increases within a specified range, convergence solutions can be obtained, and Nu/Nu_s and f/f_s will increase monotonically, in which Nu_s and f_s are Nusselt number and friction factor in the bare tube without additional volume force. This suggests that the additional volume force would increase with the increase in W_p and the flow field would achieve a better organization, which leads to better heat transfer performance. In the meantime, however, if more power is consumed, flow resistance would increase, too. This trend is demonstrated in Fig. 4, in which we can find that the effect of heat transfer optimization can achieve when Re = 200 and $W_p = 8.46\times10^{-7}$ W. At this situation, Nu number with additional volume force is beyond twice as much as that without additional volume force, while f is only increased by 1.087 times, which indicates a far greater degree of heat transfer enhancement than flow resistance increase.

Fig. 5 shows the temperature and flow fields in the cross-section of tube when Re = 500 and $W_p = 5.58\times10^{-6}$ W, the intensity of secondary flow is 0.0076% of section average flow velocity. Fig. 6 is for the case when Re = 500 and $W_p = 6.34\times10^{-6}$ W, the intensity of secondary flow is 0.017% of section average flow

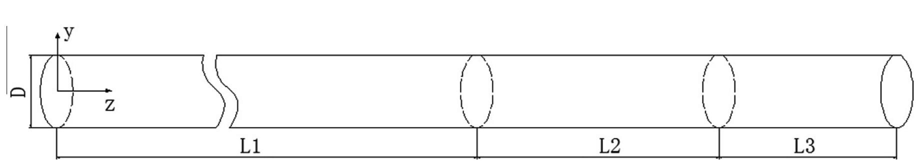

Fig. 1. The straight circular tube model.

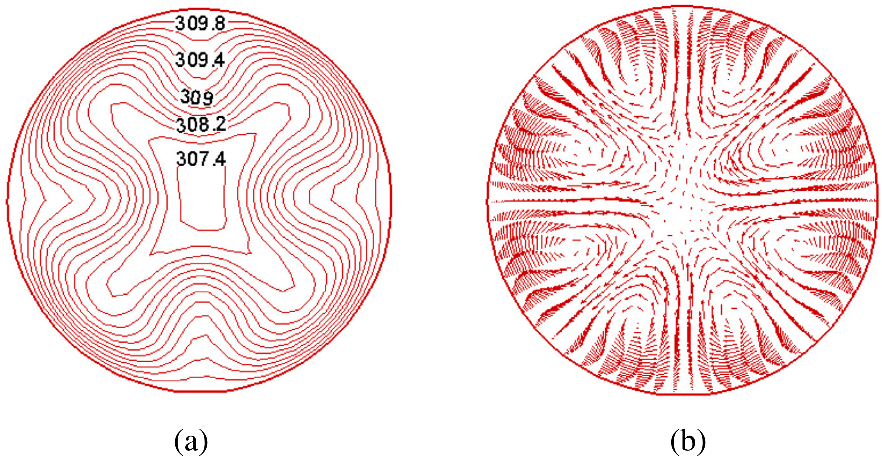

Fig. 2. The temperature field (a) and velocity field (b) in the cross-section of tube ($Re = 200$, $W_p = 8.01 \times 10^{-7}$ W).

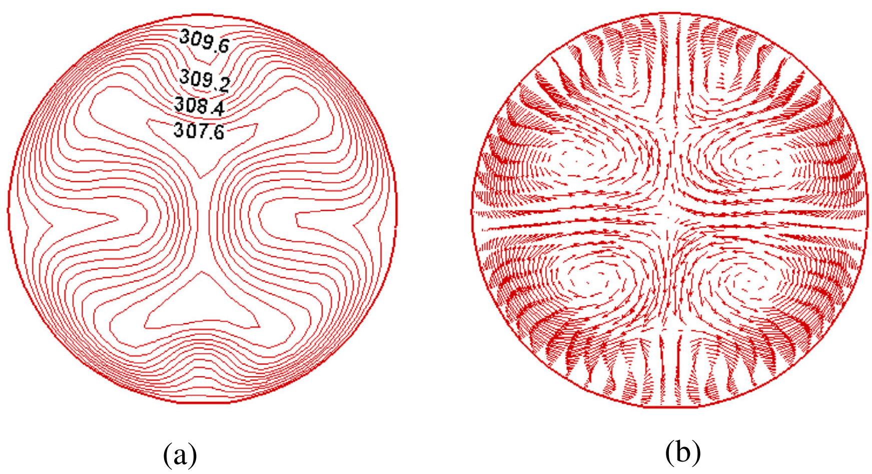

Fig. 3. The temperature field (a) and velocity field (b) in the cross-section of tube ($Re = 200$, $W_p = 8.36 \times 10^{-7}$ W).

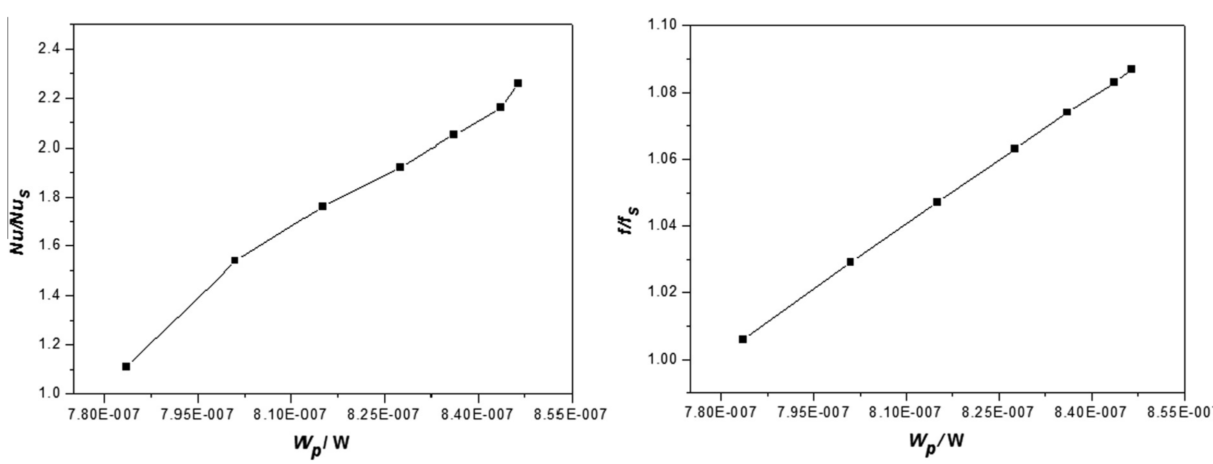

Fig. 4. Nu/Nu_s and f/f_s varying with W_p ($Re = 200$).

velocity. Fig. 7 displays the changes of *Nu* number and friction factor before and after adding the volume force when $Re = 500$. From Fig. 7, we can find that the degree of heat transfer enhancement is much greater than that of flow resistance increase when $Re = 500$ and $W_p = 6.65 \times 10^{-6}$ W. At this situation, *Nu* number with additional volume force is 2.96 times as much as that without volume force, while friction factor is only increased by 1.23 times.

We also make the calculation in other *Re*numbers. The results show that for the circular tube at a given *Re* number, when W_p varied within a specified range in which convergence solution could

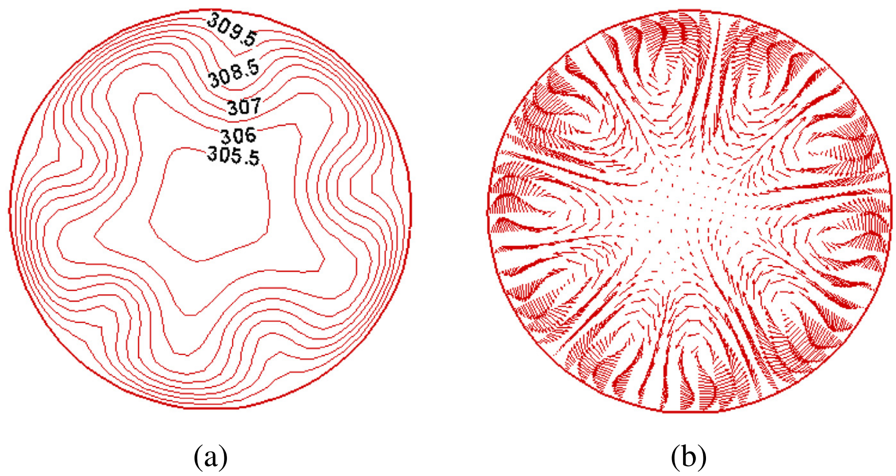

Fig. 5. The temperature field (a) and velocity field (b) in the cross-section of tube (Re = 500, W_p = 5.58 × 10^{-6} W).

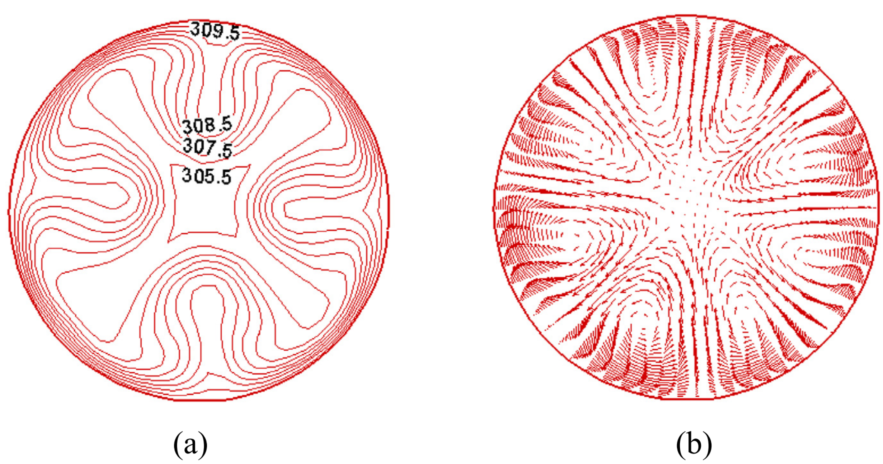

Fig. 6. The temperature field (a) and velocity field (b) in the cross-section of tube (Re = 500, W_p = 6.34 × 10^{-6} W).

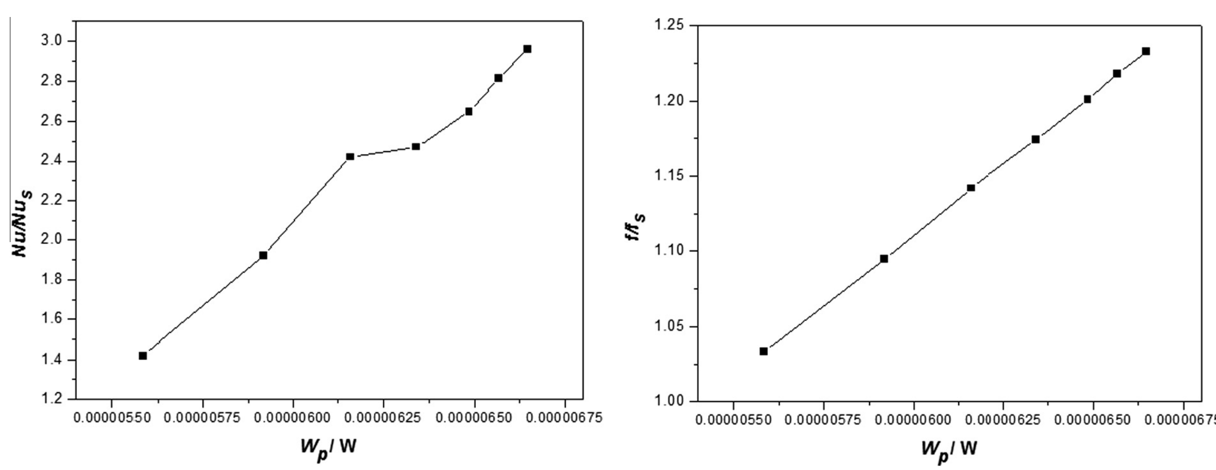

Fig. 7. Nu/Nu_s and f/f_s varying with W_p (Re = 500).

be obtained, longitudinal swirl flow with multi-vortexes in different shape and structure would be formed in the optimized velocity field, and the number and shape of the vortexes will change with W_p and Renumber. The swirl flow field with multi-vortexes would have far better heat transfer capability than a bare tube without additional volume force, and its heat transfer intensity grows much faster than flow resistance increase.

The above analysis for optimized temperature field reveals that fluid temperature gradient near tube wall is much greater than that in the central tube. Apart from this, the analysis for optimized velocity field demonstrates that the vortexes are all distributed symmetrically around central tube area, so the velocity gradient of boundary grows very small, which means that the velocity gradient near the tube wall is not increased too much. A comparison for the increment of friction factor f in Figs. 4 and 7 makes this clear. This kind of flow field is consistent with the principle of heat transfer enhancement in the core flow of tube proposed by Liu et al. [32,33], who intends to design a flow field in which the disturbance in the boundary flow is kept at the relatively lower level while the disturbance in the core flow is enhanced greatly, thereby to realize temperature uniformity in the core flow of tube. So far, when going back to review the optimization objective and the constraint conditions specified in the foregoing part of this paper, we find that the degree of entransy dissipation reflects the magnitude of thermal resistance, while power consumption reflects flow resistance. So if we want to reconcile heat transfer with fluid flow, what we need to do is to decrease thermal resistance as much as possible while keeping the flow resistance at relatively small level. In this way, the connotation of resistance deduction gains substantial embodiment.

In this paper, we have not proposed the practical way to realize the force field, the multi-vortexes flow field is maintained by the additional volume force which is a virtual force to form optimal flow field. As there are no devices inserted in the tube, it only causes the increase of less than 23% in resistance coefficient f. We think that this increase in flow resistance is caused by extra work required by viscous dissipation of the vortexes in the flow field. However, the deviation may exist, due to the formula we used in calculating resistance coefficient, which is usually used for the bare tube. In the practical applications, when some inserts are adopted to obtain the longitudinal swirl flow with multi-vortexes in a tube, the friction resistance will be much higher due to the additional inserts, and the longitudinal vortexes generated by them decay gradually in the streamwise direction due to the viscous shear force which we have not researched in this paper too. This additional resistance and the decay property of vortexes will be considered in our further study to design heat transfer units.

5. Conclusion

By taking minimum entransy dissipation as optimization objective and fixed power consumption as constraint condition, a generalized functional using Lagrange multiplier was constructed, from which optimized field equations were obtained by variational principle.

Numerical solution of flow field in the straight circular tube is analyzed. It is found that the longitudinal swirl flow with multi-vortex structure in the core flow of tube can realize excellent heat transfer performance. Compared to the bare tube, the maximal Nu number can be increased by 190%, while the friction factor f is only increased by less than 23%. It is validated that the optimization method investigated can provide theoretical guidance for designing high-efficiency and low-resistance heat transfer units of heat exchangers.

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