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A general mathematical modelling for heat and mass transfer in unsaturated porous media: an application to free evaporative cooling

W. Liu, S. W. Peng, K. Mizukami

Abstract The present paper develops a general mathematical model with some improvements in mass, momentum and energy equations, which introduce more transport mechanisms to simulate simultaneous transfer of heat and mass in the porous media unsaturated with liquid. Numerical calculation results in two-dimension are obtained for the vertical packed bed with its right opening surface exposing to atmospherical environment. The calculating data can demonstrate the cooling effect of the water evaporation for the bed if it is used as a cooling wall of building for room air-conditioning in the hot and dry climate.

Verallgemeinerte mathematische Modellierung des Wärme- und Stoffaustausches in nicht vollständig getränkten Medien
Zusammenfassung In der vorliegenden Arbeit wird ein verallgemeinertes mathematisches Modell entwickelt, wobei hinsichtlich der Impuls-, der Kontinuitäts- und der Energiegleichungen einige Verbesserungen Eingang finden, die durch Hinzufügung weiterer Transportmechanismen eine Simulation des gleichzeitigen Wärme- und Stoffaustausches in nicht vollständig getränkten porösen Medien ermöglichen. Numerische Ergebnisse für ein zweidimensional betrachtetes, vertikal gepacktes Bett mit rechtsseitig offener, in Verbindung mit der Außenatmosphäre stehender Begrenzung, werden mitgeteilt. Die Berechnungsdaten belegen den Kühlungseffekt, der durch Wasserverdunstung aus dem Bett erzielbar ist, wenn dieses in heißen und trockenen Gebieten als Kühlwand ausgebildet wird, um eine Raumklimatisierung zu bewirken.

Nomenclature

A	area, m^2
c	specific heat, J/(Kg · K)
D_l	diffusivity of water in porous materials, m^2/s
D_v	molecular diffusivity of vapor in air, m^2/s
D_{Tv}	diffusivity defined in Eq. (11), $m^2/(s \cdot K)$
D_{lv}	diffusivity defined in Eq. (12), m^2/s
g	acceleration of gravity, m/s^2
h	convective heat transfer coefficient, $W/(m^2 \cdot K)$
h_m	convective mass transfer coefficient, m/s
H	horizontal width in x direction of bed, m
k_m	apparent thermal conductivity, W/(m · K)
K_g	infiltrating conductivity of gas mixture, m/s
K_l	hydraulic conductivity of water, m/s
L	vertical height in y direction of bed, m
$\dot{m}$	mass rate of phase change per unit volume, $Kg/(m^3 \cdot s)$
m	mean vapor quantity in porous bed per unit volume, Kg/m^3
$\vec{n}$	normal vector
P	pressure, Pa
$\vec{q}_v$	vapor diffusion flux, $kg/(m^2 \cdot s)$
q_r	solar radiation, W/m^2
RH, h	relative humidity in ambient air, or in gas mixture, %
S	liquid saturation, %
t	time, s
T	temperature, K (°C)
u	velocity component in x-direction, m/s
v	velocity component in y-direction, m/s
$\vec{V}, \vec{v}$	velocity vectors, m/s
V	averaging volume, m^3
V_t	volume of porous packed bed, m^3
$\vec{w}$	velocity vector of gas-liquid interface, m/s

Greek symbols

α	tortuosity factor in Eq. (2)
β	thermal expansion coefficient, 1/K
γ	latent heat, J/Kg
ε	phase content, %
ν	kinematic viscosity, m^2/s
ρ	density, Kg/m^3
υ	mass flow factor in Eq. (2)
ϕ	porosity, %
ψ	water pressure potential, m
Φ	actual value of certain physical quantity

Subscripts

a	air, ambient
cw	cooling wall
g	gas mixture
gl	gas-liquid interface
i	inside surface
l	liquid, water

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1 Introduction

The porous materials are widely used or existing in many aspects of engineering, agriculture and environment protection because mass, momentum and energy transport in those media are closely associated with various physical phemomena and practical problems that are eagerly expected to be explained or solved. To name a few, applied examples include thermal insulation in buildings, heat pipe system coupled with earth heat source, convective drying for food and agricultural products, thermal dissipation of underground cable and direct current electrode, growing control for planting crops, disposal of thermal, chemical and nuclear wastes, and so forth.

Compared with porous media saturated with single fluid, the unsaturated bed is more sophisticated in transport processes as two-phase flow, heat and mass transfer with phase change and other complex mechanics characteristics are involved in the porous body. Although some typical theories had been established by Philip & DeVries [1, 2], Luikov [3, 4], Slattery [5, 6] and Whitaker [7, 8] and many approaches and applications, especially in evaporation (drying) processes, worked out in resent decades by authors, such as for instance Berger & Pei [9], Eckert [10], Udell [11], Vafai [12, 13], Cheng [14], etc., mathematical models need to be improved and application areas demand to be explored further more. As the study of transfer processes in the unsaturated porous media has been advancing to more and more complicated phenomena, it is necessary to adequately modify and expend the existing mathematical equations by which some applications seem not to be well explained in transport phenomena, to meet the needs of more general descriptions of physical mechanisms.

Beyond the strict theoretical systems, the way should be paved for the complex models to be solved on the bases of various practical applications. For the problem of vertical porous bed packed with unconsolidated sand unsaturated with water, the technique of volume average [5–8], in general, can be applied as multiphase system is involved in the media, but more attention should be paid to the movement relations among each flowing phase, the influence of diffusive motion of vapor on momentum transport and the solubility of equations depended on a series of coefficients of unsaturated physical properties. The principles of the irreversible thermodynamics in establishing the constructive equations [3, 4] are also of great importance, but one may encounter some difficulties in determining the phenomenon coefficients defined in the coupling equations, which included thermodynamic properties, transfer coefficients and thermogradient coefficients, etc. Hence practical applications for the model are limited.

In the present paper, we have connected our theoretical analysis to free evaporating cooling in unsaturated porous packed bed that works as a wall (or part of it) in houses or buildings, which is so far seldom considered theoretically by researchers through our searching for literature. This method of cooling may be used in designing a useful air-conditioning device that could be installed in private houses and public buildings to transfer heat flux from room to ambient air, thereby reducing the temperature of room by its free evaporative effect in the porespaces and on the surface exposed directly to the outside atmosphere. Remarkable advantage in acceptance of this simple device, instead of conventional air-conditioner with thermodynamic cycle, comes to that no electricity, first, is consumed to drive flowing media, only water energy at low grade, second, is needed as its energy source and moving parts, third, are not necessarily involved when operating it. The study of cooling performances for the vertical sand bed working at the hot and dry summer, therefore, is of great importance for energy conservation.

2 Basic consideration

Before the overall mathematical model for the physical phenomena mentioned above to be developed, a basic consideration might be presented first: For the momentum transport in unsaturated porous media, are motion or migration mechanisms of the multiphase fluids dominated uniquely by Darcy's law or extended Darcy's law? To discuss this question, some of our considerations for the model analysis are expressed as follows.

In modelling the phenomena processes in the unsaturated porous media, one is immediately confronted with the choice between two kinds of momentum equations: One is based Darcy's law and another is analogous to Navier-Stokes equation. Many authors preferred to construct or introduce the momentum equations for liquid- and gas-phase (vapor-air mixture) according to force balance by assuming that convective and diffusive movements of each phase and vapor are so small, that acceleration and inertial effects of fluids are negligible. In addition, under the zero assumption for momentum change, only pressure gradient with gravity potential and viscosity force are in balance with each other (plus capillary potential for the liquid phase equation), and as the result, momentum equations or explicit expressions for velocities (so-called motion equations) in the form of unsaturated Darcy's law are obtained. However, this methodology seems to convince people that Darcy mechanism plays a dominated and unique role for the momentum transfer in unsaturated problem, and places all difficulties of uncertainties to the physical property coefficient K, unsaturated permeability.

A well-established expression from analogy with Navier-Stokes equation is more general and suitable than Darcy's equation for the mentioned problem especially in the theory and application of convective drying. We could find several examples to prove this proposition. First of all, if diffusive motion of vapor makes a big contribution to mass and energy conversation, it must have done the same thing in the momentum transfer of fluids. But this action could not be found in the Darcy's momentum or motion equations. Secondly, under the circumstances of larger temperature gradient, natural convection of gas-phase fluid is much obvious in the porous packed bed and there exists a relatively larger Ra number in the system. This requests that inertial or convective terms should be added into momentum equation. Thirdly, when the investigation is focused on the application problem with higher Da number, the gravity is bigger than capillary force and the status of hydrostatic equilibrium may be hard to be satisfied in the process of nonisothermal infiltration of fluids. Finally, Darcy's law, on the one hand, was originally derived on the base of liquid-saturated porous medium. Thus any kind of extensions for it in the unsaturated porous medium may cause relatively larger deviation, due to that Darcy's law, in this case, may decay to one of the motive multimechanisms that control the momentum motion of fluids by assemble effects in stead of uniquely depending on Darcy's phenomena. On the other hand, the reduction of general momentum equations for multiphase flow in the unsaturated porous medium to Darcy's law is only regarded as a case of simplification which certainly exerts a series of assumptions and restraints on the mathematical descriptions [15].

3 Mathematical model

We begin our model developments with some basic assumptions that allow us to use Whitaker's theory and remain the useful and important terms in the partial differential equations.

(i) Homogeneous and isotropic porous media with no distension and contraction is considered.
(ii) Local thermal equilibrium is satisfied throughout the porous media.
(iii) Liquid film in the form of free water enclosing solid-matrix is in funicular (continuous) state and the migration mechanism related to absorptive water is not taken into account.
(iv) Gaseous mixture (air plus vapor) in the porespace is also in funicular state and treated as the ideal gas.
(v) The Boussinesq's approximation is valid for natural convection in the space of gas-phase.
(vi) All dispersion terms associated with the deviations from the average values in transport equations can be considered negligible.
(vii) Fluids in the porous media are incompressible and the dispassion term in the energy equation is dropped.

In general, the transfer phenomena in porous media can be described in terms of the volume-averaged quantities. Each of the volume-averaged quantity $\langle \Phi \rangle$ is defined usually by

$$\langle \Phi \rangle = \frac{1}{V} \iiint_V \Phi \, \mathrm{d}V \tag{1}$$

where Φ is the actual value of certain physical quantity at a particular point inside the average volume V, and all volume-averaged quantities used in this paper, such as temperature T, velocity components u and v, liquid content ε_l, phase change rate of liquid per unit volume $\dot{m}$, and so on, are defined in the same manner as $\langle \Phi \rangle$ in Eq. (1) to simplify the equation expressions.

3.1 Transport of vapor phase

If the phase change of liquid takes place in the porous media, diffusion motion of vapor influences the transport processes of mass, momentum and energy with no doubt. For the sake of simplicity in the mathematical description, one may introduce the governing equations according to continuum mechanics without involving diffusive mechanism [16]. Another simplified method is just adding the diffusive effect of vapor into continuity and energy equations without considering it in momentum equation [7]. Philip and DeVries [1, 2] developed a coupling model, similar to Luikov's equations [3], by paying much attention to the vapor diffusion and combining vapor diffusion with capillary diffusion of water in porous media, but no momentum movement was taken into account. The most simple way, to calculate the diffusive velocity of the vapor phase, is to make use of the Stefan diffusion model through a stagnant gas film [17], and through it the bulk velocity of gas phase could be estimated by supposing that air is inert in the porespace in the meantime [18]. In the present investigation, we hope to combine the diffusive motion of vapor with momentum motion of gaseous mixture, since any moving mass with an extended velocity can cause momentum change between two neighboring points in the averaging volume, no matter how it is driven by what kind of forces.

The equation of the vapor diffusion, after considering the vapor flux resulting from the bulk flow of gas mixture, can be written as

$$\vec{q}_v = -D_v \, \upsilon \alpha \varepsilon_a \nabla \rho_v \tag{2}$$

where $\upsilon\,[=P/(P-p_v)]$ is the mass flow factor reflecting the bulk flow, α represents the tortuosity factor, ε_a designates the volumetric air content and $\varepsilon_a \cong \varepsilon_g = \phi - \varepsilon_l$ for approximation. Note that ϕ and ε_l stand for the porosity and the liquid content respectively, we rewrite Eq. (2) to

$$\vec{q}_v = -D_v \, \alpha(\phi - \varepsilon_l) P/(P - p_v) \nabla \rho_v \tag{3}$$

From the reference [1, 2], we have

$$\rho_v = \rho_{vs} h = \rho_{vs} e^{\psi g/RT} \tag{4}$$

Differentiating Eq. (4) yields

$$\nabla \rho_v = h \nabla \rho_{vs} + \rho_{vs} \nabla h \tag{5}$$

In Eq. (4), taking note of the relative humidity $h = f(\varepsilon_l, T)$, we can write

$$\nabla h = \frac{\partial h}{\partial \varepsilon_l} \nabla \varepsilon_l + \frac{\partial h}{\partial T} \nabla T \tag{6}$$

Substitution of Eq. (6) into (5) leads to

$$\nabla \rho_v = h \frac{\mathrm{d}\rho_{vs}}{\mathrm{d}T} \nabla T + \rho_{vs} \left(\frac{\partial h}{\partial \varepsilon_l} \nabla \varepsilon_l + \frac{\partial h}{\partial T} \nabla T \right) \tag{7}$$

By using Eq. (4), one can obtain the partial differentials in the parentheses of right hand side of Eq. (7) as

$$\frac{\partial h}{\partial \varepsilon_l} = \frac{\partial h}{\partial \psi} \frac{\partial \psi}{\partial \varepsilon_l} = \frac{hg}{RT} \frac{\partial \psi}{\partial \varepsilon_l} \tag{8}$$

and

$$\frac{\partial h}{\partial T} = -\frac{hg\psi}{RT^2} \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (7) first, and Eq. (7) into Eq. (2) then, by letting $\vec{V}_v = \vec{q}_v/\rho_v$, one can obtain an equivalent velocity for vapor diffusion motion

$$\vec{V}_v = -D_{Tv} \nabla T - D_{lv} \nabla \varepsilon_l \tag{10}$$

The diffusive coefficients in Eq. (10) take the form of

$$D_{Tv}=D_v\,\alpha(\phi-\varepsilon_l)\,\frac{P}{P-p_v}\left(\frac{1}{\rho_{vs}}\frac{\mathrm{d}\rho_{vs}}{\mathrm{d}T}-\frac{g\psi}{RT^2}\right) \tag{11}$$

and

$$D_{lv}=D_v\,\alpha(\phi-\varepsilon_l)\,\frac{P}{P-p_v}\,\frac{g}{RT}\,\frac{\partial\psi}{\partial\varepsilon_l} \tag{12}$$

In above coefficient relations, molecular diffusion coefficient D_v can be easily evaluated from Philip and DeVries' paper [1] and density of saturated vapor from Mayhew and Rogers' book [19]. In addition, the water pressure potential ψ is the function of water content ε_l and temperature T: $\psi=f(\varepsilon_l, T)$. However, for the medium sand, it may approximately have the form [20]

$$\psi=-2.41-0.002\,\varepsilon_l^{-1.75} \tag{13}$$

3.2
Governing differential equations

With the assumptions mentioned earlier, we can present the overall partial differential equations in two-dimension for unsteady state as follows.

3.2.1
Continuity equations

$$\frac{\partial(\rho_l\varepsilon_l)}{\partial t}+\frac{\partial}{\partial x}(\rho_l\varepsilon_l u_l)+\frac{\partial}{\partial y}(\rho_l\varepsilon_l v_l)=-\dot{m} \tag{14}$$

$$\frac{\partial(\rho_g\varepsilon_g)}{\partial t}+\frac{\partial}{\partial x}(\rho_g\varepsilon_g u_g)+\frac{\partial}{\partial y}(\rho_g\varepsilon_g v_g)=\dot{m} \tag{15}$$

$$\frac{\partial(\rho_v\varepsilon_g)}{\partial t}+\frac{\partial}{\partial x}\left(\rho_v\varepsilon_g\left(u_g-\left(D_{Tv}\frac{\partial T}{\partial x}+D_{lv}\frac{\partial\varepsilon_l}{\partial x}\right)\right)\right)+\frac{\partial}{\partial y}\left(\rho_v\varepsilon_g\left(v_g-\left(D_{Tv}\frac{\partial T}{\partial y}+D_{lv}\frac{\partial\varepsilon_l}{\partial y}\right)\right)\right)=\dot{m} \tag{16}$$

3.2.2
Momentum equations

$$\frac{\partial u_l}{\partial t}+u_l\frac{\partial u_l}{\partial x}+v_l\frac{\partial u_l}{\partial y}-\frac{\dot{m}}{\rho_l\varepsilon_l}u_l=-\frac{gD_l}{K_l}\frac{\partial\varepsilon_l}{\partial x}-\frac{g\varepsilon_l}{K_l}u_l-\frac{g\varepsilon_g}{K_g}(u_l-u_g)+\nu_l\left(\frac{\partial^2u_l}{\partial x^2}+\frac{\partial^2u_l}{\partial y^2}\right) \tag{17}$$

$$\frac{\partial v_l}{\partial t}+u_l\frac{\partial v_l}{\partial x}+v_l\frac{\partial v_l}{\partial y}-\frac{\dot{m}}{\rho_l\varepsilon_l}v_l=-\frac{gD_l}{K_l}\frac{\partial\varepsilon_l}{\partial y}-\frac{g\varepsilon_l}{K_l}v_l-\frac{g\varepsilon_g}{K_g}(v_l-v_g)+\nu_l\left(\frac{\partial^2v_l}{\partial x^2}+\frac{\partial^2v_l}{\partial y^2}\right)-g \tag{18}$$

$$\frac{\partial u_g}{\partial t}+u_g\frac{\partial u_g}{\partial x}+v_g\frac{\partial u_g}{\partial y}+\frac{\dot{m}}{\rho_g\varepsilon_g}u_g-\frac{\dot{m}}{\rho_g\varepsilon_g}\left(D_{Tv}\frac{\partial T}{\partial x}+D_{lv}\frac{\partial\varepsilon_l}{\partial x}\right)=-\frac{1}{\rho_g}\frac{\partial P}{\partial x}-\frac{g\varepsilon_g}{K_g}(u_g-u_l)+\nu_g\left(\frac{\partial^2u_g}{\partial x^2}+\frac{\partial^2u_g}{\partial y^2}\right) \tag{19}$$

$$\frac{\partial v_g}{\partial t}+u_g\frac{\partial v_g}{\partial x}+v_g\frac{\partial v_g}{\partial y}+\frac{\dot{m}}{\rho_g\varepsilon_g}v_g-\frac{\dot{m}}{\rho_g\varepsilon_g}\left(D_{Tv}\frac{\partial T}{\partial y}+D_{lv}\frac{\partial\varepsilon_l}{\partial y}\right)=-\frac{1}{\rho_g}\frac{\partial P}{\partial y}-\frac{g\varepsilon_g}{K_g}(v_g-v_l)+\nu_g\left(\frac{\partial^2v_g}{\partial x^2}+\frac{\partial^2v_g}{\partial y^2}\right)-g\beta(T-T_w) \tag{20}$$

3.2.3
Energy equation

$$\begin{aligned}&\frac{\partial}{\partial t}((\rho c)_mT)+\rho_l\varepsilon_l c_l\left(u_l\frac{\partial T}{\partial x}+v_l\frac{\partial T}{\partial y}\right)+\varepsilon_g c_a\left(u_g\frac{\partial}{\partial x}(\rho_aT)+v_g\frac{\partial}{\partial y}(\rho_aT)\right)\\&+\varepsilon_g c_v\left(\left(u_g-\left(D_{Tv}\frac{\partial T}{\partial x}+D_{lv}\frac{\partial\varepsilon_l}{\partial x}\right)\right)\frac{\partial}{\partial x}(\rho_vT)+\left(v_g-\left(D_{Tv}\frac{\partial T}{\partial y}+D_{lv}\frac{\partial\varepsilon_l}{\partial y}\right)\right)\frac{\partial}{\partial y}(\rho_vT)\right)\\&+\rho_lc_lT\left(u_l\frac{\partial\varepsilon_l}{\partial x}+v_l\frac{\partial\varepsilon_l}{\partial y}\right)+\rho_gc_aT\left(u_g\frac{\partial\varepsilon_g}{\partial x}+v_g\frac{\partial\varepsilon_g}{\partial y}\right)\\&+\rho_vc_vT\left(\left(u_g-\left(D_{Tv}\frac{\partial T}{\partial x}+D_{lv}\frac{\partial\varepsilon_l}{\partial x}\right)\right)\frac{\partial\varepsilon_g}{\partial x}+\left(v_g-\left(D_{Tv}\frac{\partial T}{\partial y}+D_{lv}\frac{\partial\varepsilon_l}{\partial y}\right)\right)\frac{\partial\varepsilon_g}{\partial y}\right)\\&=\frac{\partial}{\partial x}\left(k_m\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_m\frac{\partial T}{\partial y}\right)-\dot{m}\cdot\gamma\end{aligned} \tag{21}$$

In the energy Eq. (21), apparent heat capacity and thermal conductivity are

$$(\rho c)_m=\varepsilon_s\rho_sc_s+\varepsilon_l\rho_lc_l+\varepsilon_g\rho_gc_g \tag{22}$$

and

$$k_m=\varepsilon_sk_s+\varepsilon_lk_l+\varepsilon_gk_g \tag{23}$$

and a limiting constraint that is coupled with the above system of equations is

$$\varepsilon_g=1-\varepsilon_s-\varepsilon_l \tag{24}$$

3.3
Model analysis and discussion

There are, for the porous packed bed, eight field quantities (T, P, ε_l, $\dot{m}$, u_l, v_l, u_g, v_g) in the above governing Eqs. (14–21) and the first four are related to Luikov's model in which a general moisture θ was defined to represent the liquid content ε_l and the vapor content ε_v depending on the evaporative rate of liquid $\dot{m}$ that can be defined theoretically by [7]

$$\dot{m} = -\frac{1}{V} \int_{A_{gl}} \rho_g (\vec{v}_g - \vec{w}) \cdot \vec{n}_{gl} \, \mathrm{d}A \tag{25}$$

If $\dot{m}$ takes the negative value, the transport behaviors stated before will be of the condensation characteristics, too. The present model, therefore, can predict either evaporation or condensation problems. The vapor generation of phase change in the unsaturated porous body V_t can be evaluated by the integral

$$m = \frac{1}{V_t} \int_{V_t} \dot{m}(x, y, z, t) \, \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}t \tag{26}$$

The influence of vapor diffusion on the inertia of gas phase is demonstrated in the momentum Eqs. (19, 20) through the equivalent velocity defined in Eq. (10) which is also emerging in the continuity and energy equations. Thus, the existing state (liquid and vapor) and the moving manner (infiltration and diffusion) for the moisture in the porous media have been simultaneously reflected in the overall governing equations.

For the gas-phase flow in the porous space, a Darcy resistance of gaseous mixture is introduced in momentum equations and gaseous infiltrating conductivity K_g, corresponding to liquid hydraulic conductivity K_l, is defined to demonstrate the blocking mechanism of porous solid skeleton filmed with free liquid to infiltrating flow of the mixture. To obtain the calculating data for this coefficient, we have established an expression between K_g and K_l, according to Kozeny and Ergun's theory [21] by supposing that solid particles in the porous matrix are in the shape of sphere, as follows

$$K_g(S) = (1-S)^3 \left(\frac{1-\phi}{1-\phi(1-S)} \right)^{4/3} \frac{\nu_l}{\nu_g} K_l(S) \tag{27}$$

Where the value of $K_l(S)$ can be estimated from the experimental curves of Jury [23] for unsaturated sand. With the relation (27), we can also add an important term, the third one on the right hand side of Eqs. (17, 18), in terms of the flowing resistance that indicates the reaction of gaseous mixture on liquid, considering the relative motion between the two fluids.

In the modelling development, Dalton's partial pressure law, for gas phase species (vapor and air), is regarded to be valid throughout gas-phase space under the consideration of continuum, which allows that a single symbol ε_g for the gaseous mixture content could appear in the continuity, momentum and energy equations, instead of ε_v for vapor and ε_a for air. Thus, the difficulty of establishing the overall model for the multiphase system decreases since just one content variable ε_l is involved in the governing equations, and ε_g is a dependent variable.

Under the circumstances of simplification for the overall model, one can simply let $\dot{m}=\rho_v=0$ except in the continuity equations of liquid and gas, to neglect the influences of evaporation and condensation on the momentum and energy transport. By doing so, the above mathematical model can be reduced to a great extent.

3.4
Model application to free evaporation

In the problem under investigation, a porous packed bed filled with unconsolidated sand is vertically designed as a thin cube [23, 24] with a metal plate as its left boundary wall having no liquid-permeation, which contacts the room air, and an uncovered surface as its right opening side. Water is filled into the bed from the left boundary surface. The free opening surface is exposed to ambient air, in which relative humidity, temperature, wind velocity and other climate conditions may change with days, and the left surface made by metal may be acting as a wall of the room, on which heat transfer enhancements, for instance a surface with fins or attached by a fan to yield convective air flow, could be considered. When it is driven by the meteorological data, all field gradients of physical quantities in the packed bed, such as temperature, water content, velocities of liquid and gas mixture, pressure etc., begin to change, and the heat transfer and the moisture migration will keep going on at various rates under different ambient conditions.

As the results of water evaporation on the free surface, the temperature on the right boundary decreases to the lowest level for the porous layer. Inside the packed bed, thermal gradient influences moisture transfer that also affects heat flux in return, but the main driving forces for moisture motion are from capillary and molecular diffusion mechanisms, which help to bring evaporating effect on the free boundary surface to the interior of the sand bed. So ambient relative humidity plays an important role in determining water infiltrating flow and water vapor diffusion. The temperature difference between the metal plate and the room air could reach a significant level, which generates the cooling effect for the room, depending on the ambient conditions.

For the problem of the free evaporative cooling, the corresponding boundary conditions for the vertical porous bed matching the present governing equations in the steady-state could be written as

$$X = 0: \quad \begin{aligned} &\varepsilon_l = \phi \\ &u_l = v_l = u_g = v_g = 0 \\ &-k_m \frac{\mathrm{d}T}{\mathrm{d}x} = h_i (T_{rm} - T) \end{aligned} \tag{28}$$

$$x = H: \quad \begin{aligned} &P = P_a \\ &u_l = v_l = u_g = v_g = 0 \\ &-k_m \frac{\mathrm{d}T}{\mathrm{d}x} = h_o (T - T_a) + h_m (\rho_{vs} - \rho_{v\infty}) \gamma - q_r \end{aligned} \tag{29}$$

$$y = 0: \quad \begin{aligned} &u_l = v_l = u_g = v_g = 0 \\ &\frac{\partial T}{\partial y} = 0 \end{aligned} \tag{30}$$

$$y = L: \quad \begin{aligned} &u_l = v_l = u_g = v_g = 0 \\ &\frac{\partial T}{\partial y} = 0 \end{aligned} \tag{31}$$

4
Results and discussion

Under the boundary conditions (28–31), two-dimensional, steady-state, numerical computations for the present mathematical equations have been carried out by finite difference method and the SIMPLER scheme has been adopted to solve the gas-phase momentum equations which are nonlinear and pressure-related. The underrelaxation factors are chosen for every variable to eliminate the perturbation of variables and avoid divergence of iteration due to nonlinearity of the equations, which can also ensure that every coupled quantity well matches with each other in convergence rate. The calculating process is regarded to get to a convergent solution only if the temperature difference of inner nodes between two iterative steps is less than 0.001 °C.

The reference values for ambient parameters in our calculation are on the basis of common climate conditions in the summer and include a constant coefficient of forced convective heat transfer in the left boundary: RH=60%, T_a=35 °C, V_a=3 m/s and h_i=10 W/(m^2 · °C). The calculating height and width of the evaporative cooling bed are selected by L=200 mm and H=25 mm. The calculating results in this paper do not include the influences of variations of ambient and constructive parameters on the cooling effects.

As illustrated in Fig. 1, the temperature in the porous packed bed is below the ambient temperature due to water evaporation both in the bed and on the free evaporative surface and a meaningful effect of temperature-dropping (about 5 °C) on the left surfaces can be observed. The data from temperature contour lines can show the potential of the evaporative bed for serving as an air-conditioner that is taking out the heat flux from the room at a relatively lower rate but without changing the relative humidity of the room air, if the working surface made by metal is designed to be big enough.

Figure 2 indicates the change of water content in the vertical bed with a constant water supply along the whole left-boundary surface. It has well explained the mechanism of movement and force-action of fluids in the porous bed, that is, under the coupling actions of gravity, capollary force and free surface evaporation, the direction of main migration for moisture transfer is from the left-bottom area to the right-upper area. The temperature contour lines in Fig. 1 supports this explanation too. All other forces, such as viscous force, Darcy's resistance, interaction force between gas and liquid, and so on, are less important, but dropping them from the overall equations will surely change the field distributions.

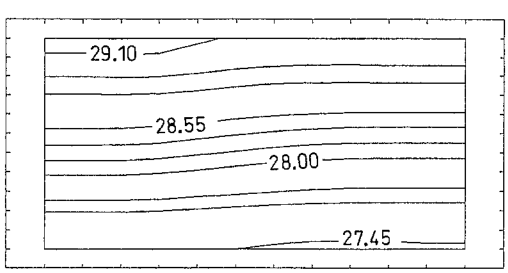

Fig. 1. Temperature contour line in porous bed (°C)

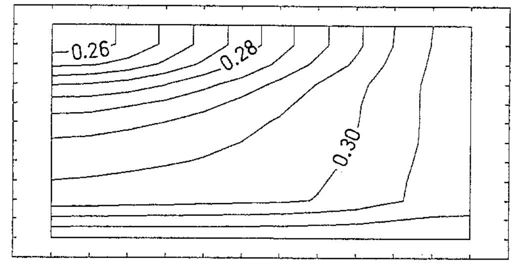

Fig. 2. Water content contour line in porous bed (%)

The contour of rate of water evaporation in the packed bed is diagrammed in Fig. 3 that clearly indicates the evaporative behavior of moisture in the porosity. It also obviously shows that the evaporation on the free right surface is much stronger than that in the interior of the vertical bed (more than an order of magnitude). But in the inner bed, relatively heavy evaporation occurs in the right-upper area and vapor condensation emerges in the left-bottom area. This proves again that the migration process both for heat and mass has a regularity of diagonal direction.

From Fig. 4, we can see a convective roll of gas mixture in clockwise direction demonstrated by the contour lines of stream function. It implies that in the upper region of flow field, the movement of mixture positive to x-axis plays a role to help the moisture to be transferred from the inside to the outside through the porous bed, but in the lower part of it, there exists an opposite action because the direction of mixture movement differs from that of moisture migration. This also says that how important the convection of gas mixture could be to influence the other field quantities.

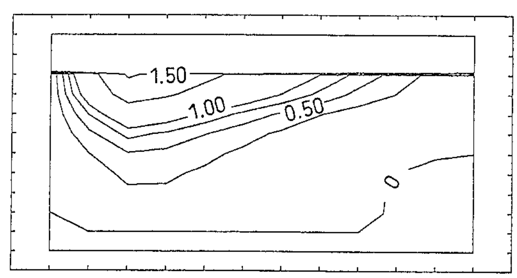

Fig. 3. Evaporative rate contour line in porous bed (Kg/(m^3–s)×10^6)

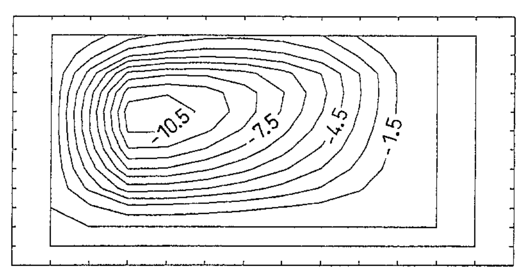

Fig. 4. Stream function contour line in porous bed (×10^{10})

5
Conclusion

The improvements are made for the governing equations describing simultaneous heat and mass transfer in the unsaturated porous media by considering the influences of diffusive motion of vapor on transport of mass, momentum and energy, and regarding the mechanisms of resistance and movement between liquid and gaseous mixture. The overall equations with eight variables are of not only the theoretical completeness, but also the numerical solubility.

The field data for a vertical packed bed with a free evaporative surface are obtained numerically, which can show the potential of the unsaturated sand bed working as a device for the room air-conditioning, which surely has obvious advantages in the sense of energy-saving.

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