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Ecological optimization and coefficient of performance bounds of general refrigerators

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HIGHLIGHTS

- The heat exchanging processes are non-isothermal.
- The internal dissipations are considered.
- The ecological criterion is adopted to optimize the refrigerators.
- General theoretical COP bounds have been deduced.
- The COP bounds of different kinds of refrigerators have been proposed.

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ABSTRACT

An analysis of COP and its bounds at maximum ecological criterion for general refrigerators is conducted. For generality, both the non-isothermal heat transfer processes and the internal dissipations are considered. Under different situations, the COP under the maximum ecological criterion have been studied systematically. And the general upper and lower bounds of the optimal COP have been obtained. Furthermore under maximum ecological criterion, the COP of general endoreversible refrigerators have also been studied. And the COP bounds of different kinds of refrigerators have been analyzed. As actual refrigerators may not operate under the condition of maximum COP or maximum cooling load, but operate under the maximum ecological condition which indicates the best compromise between the refrigeration rate and the loss of refrigeration rate. This paper could provide a practical insight for designing and operating actual refrigerators.

1. Introduction

Conditioned on energy saving and fuel depletion, the optimization of real thermodynamical cycles has attracted rising attention. As we all know, the real refrigerator can never achieve the Carnot coefficient of performance. And the realization of the Carnot refrigerator leads to vanishing cooling load rate since in the ideal Carnot refrigerator cycle, all the processes are quasistatic and infinitely slow. Finite time thermodynamic analysis has provided a perspective to solve this dilemma.

For heat engines, maximum power output is the main optimization criterion, and many efforts have been devoted to optimize classical and quantum heat engines under that criterion [1–7]. However for refrigerators, the minimum power input is not an appropriate optimization criterion [8], and much research have been focused on selecting figure of merits for optimizing refrigerators by maximizing the per-unit-time COP, Velasco et al. [9] obtained the upper bound of COP, $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, i.e. the CA coefficient of performance, for endoreversible refrigerators with $\varepsilon_C = T_c/(T_h - T_c)$ being

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the Carnot COP, where T_c and T_h are the temperatures of the cold and hot reservoirs, respectively. Furthermore under the maximum cooling power criterion, Apertet et al. [10] studied the endoreversible and exoreversible refrigerators and claimed that the real-life working conditions of the refrigerators do not correspond to a maximum cooling. In addition, Yan and Chen [11] conducted the optimization with the target function $\varepsilon\dot{Q}_c$ where $\dot{Q}_c$ is the cooling load rate of the refrigerators. To step further, de Tomas et al. [12] introduced the unified optimization criterion $\chi = zQ_{in}/t_{cycle}$ both for heat engines and refrigerators, where z is the converter efficiency (η for heat engines and ε for refrigerators), Q_{in} is the heat absorbed by the system and t_{cycle} denotes the time duration for a cycle. By taking χ as the target function, based on the low dissipation model, Wang et al. [8] proposed that the COP at maximum χ was bounded between 0 and $(\sqrt{9+8\varepsilon_C}-3)/2$. Besides, through the minimally nonlinear irreversible refrigerator model, Izumida et al. [13] also obtained the same bounds as those in Ref. [8] under the tight coupling condition. In addition, Allahverdyan et al. [14] also investigated quantum refrigerators and obtained some new bounds of COP under the χ figure of merit.

Actual refrigerators may work under a compromise between energy benefits and losses. Hernández et al. [15] proposed a new figure of merit, accounting for both the energy benefits and losses. It is defined as $\Omega = (2\varepsilon - \varepsilon_C)W$ where ε and W are the COP and the work input to the refrigerators. Based on the Ω criterion, de Tomas et al. [16] and Long et al. [17] obtained the COP of refrigerators through the low dissipation model and the minimally nonlinear irreversible model. Furthermore, considering the impact of the environment, Angulo proposed the ecological criterion for heat engines [18], and Yan [19] developed the ecological criterion for the refrigerators, that is $E = \dot{Q}_c - \lambda T_0\dot{\sigma}$, where $\dot{\sigma}$ is the entropy generation rate, λ is the dissipation coefficient of the cooling load, and T_0 is the temperature of the ambient. This figure of merit indicates the best compromise between the refrigeration rate and the loss of refrigeration rate. Many researches had been focused on the irreversible Carnot refrigerators under the ecological optimization criterion [20–22].

However in the traditional models, very few literatures account for the non-isothermal heat exchanging processes [23–28]. In the linear irreversible and minimally nonlinear irreversible models, we donot to specify the concrete cycle processes, however these models are endoreversible, and do not consider the internal dissipations. For generality, the non-isothermal heat exchanging processes and the internal dissipations are both considered. In this paper, we have systematically studied the COP of the general refrigerators at the maximum ecological criterion, and then the lower and upper bounds of the COPs have been proposed. In addition, the COPs of general endoreversible refrigerators under the ecological figure of merit are also considered. And the COP bounds of different kinds of refrigerators have been analyzed. Finally some concluding remarks are given.

2. Mathematical model

As to refrigerators, the cooling load Q_c is absorbed from the cold reservoir (T_c), and a certain amount of heat Q_h, is evacuated to the hot reservoir (T_h) at the end of a cycle. The heat transfer law between the heat source and the working substance is assumed to conform to Newton's law of cooling:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = cm\frac{\mathrm{d}T}{\mathrm{d}t} = k(T_s - T) \tag{1}$$

where c is the heat capacity, m is the working substance mass, T is the working substance temperature, T_s is heat source temperature, k is the heat conductance. The initial temperature of the working substance is T_{c0} and T_{h0}, at the beginning of the cooling and heating processes, respectively. According to Eq. (1), the working substance temperature in the heat absorbing process is a function of time t:

$$T = T_c - (T_c - T_{c0})\mathrm{e}^{-t/\sum_c} \tag{2}$$

where $\sum_c = c_c m/k_c$. The time duration of the heat absorbing process is τ_c, the cooling load can be calculated as:

$$Q_c = \int k_c(T_c - T)\mathrm{d}t = c_c m(T_c - T_{c0})\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right). \tag{3}$$

The relative entropy change of the working substance in the heat absorbing process is given by:

$$\Delta s_c = \int \frac{\mathrm{d}Q_c}{T} = c_c m \ln\frac{T_c - (T_c - T_{c0})\mathrm{e}^{-\tau_c/\sum_c}}{T_{c0}}. \tag{4}$$

Similarly, the heat evacuated to the hot reservoir and the entropy change during the heat releasing process are given by

$$Q_h = c_h m(T_{h0} - T_h)\left(1 - \mathrm{e}^{-\tau_h/\sum_h}\right) \tag{5}$$

$$\Delta s_h = -c_h m \ln\frac{T_h + (T_{h0} - T_h)\mathrm{e}^{-\tau_h/\sum_h}}{T_{h0}} \tag{6}$$

where $\sum_h = c_h m/k_h$, τ_h is the time duration of the heat releasing process. In this paper, we assume that the compressing and expanding processes proceed instantaneously and the time for completing those processes is zero. In order to quantify the

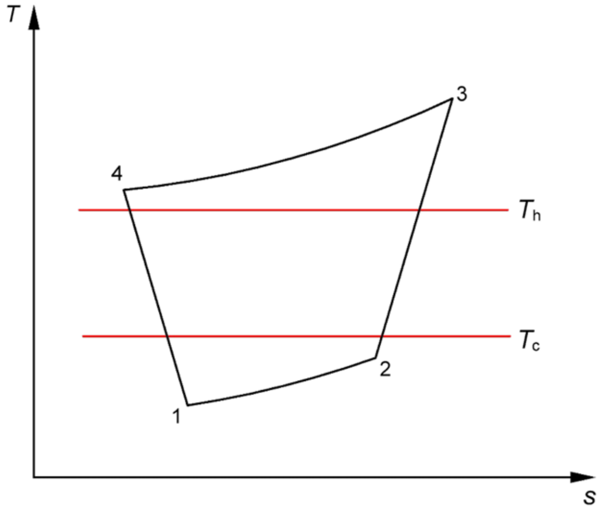

Fig. 1. The schematic T–S diagram of the general refrigerator model with internal dissipations. In process 1–2, the working medium absorbs heat from the cold reservoir in the duration τ_c. Meanwhile the temperature increases. In process 3–4, the working medium releases heat to the hot reservoir in the duration τ_h. And the temperature decreases. As the internal dissipations have been considered, the compression and expansion processes are no longer isoentropic.

effect of the internal dissipations of the working fluid on the performance of the refrigerator, we can introduce a parameter $I_s = \Delta s_h / \Delta s_c$ [29,30]. It can characterize fully the degree of internal irreversibility resulting from the working fluid. When $I_s = 1$, the internal dissipations are vanished. When $I_s > 1$, the refrigerator is internally irreversible. Therefore the model studied in present paper is more general and realistic since it accounts for both the non-isothermal heat the exchanging processes and the internal dissipations. The T–S diagram of present model is shown in Fig. 1. Combining Eqs. (4) and (6) with I_s, we have

$$\frac{T_c - (T_c - T_{c0})\mathrm{e}^{-\tau_c/\sum_c}}{T_{c0}} \left(\frac{T_h + (T_{h0} - T_h)\mathrm{e}^{-\tau_h/\sum_h}}{T_{h0}} \right)^{\gamma/I_s} = 1 \tag{7}$$

where $\gamma = c_h/c_c$. The entropy production is

$$\sigma = \frac{Q_h}{T_h} - \frac{Q_c}{T_c}. \tag{8}$$

The coefficient of performance ε is

$$\varepsilon = \frac{Q_c}{Q_h - Q_c}. \tag{9}$$

According to Refs. [19,31], λ is equivalent to the COP of a reversible Carnot refrigerator, therefore the ecological criterion $\dot{E} = \dot{Q}_c - \lambda T_0 \dot{\sigma}$ can be rewritten as $\dot{E} = \dot{Q}_c - \varepsilon_C T_0 \dot{\sigma}$. Thus

$$\dot{E} = \frac{Q_c - \varepsilon_C T_0 (Q_h/T_h - Q_c/T_c)}{\tau_h + \tau_c}. \tag{10}$$

Combining Eqs. (7) and (10), and letting $\partial \dot{\Omega} / \partial T_{c0} = 0$, we have,

$$\frac{T_c + \varepsilon_C T_0}{\varphi^2} - \varepsilon_C T_0 I_s \frac{\left(1 - \mathrm{e}^{-\tau_h/\sum_h}\right)^2 \left[\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)\varphi + \mathrm{e}^{-\tau_c/\sum_c}\right]^{-I_s/\gamma - 1}}{\left\{\left[\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)\varphi + \mathrm{e}^{-\tau_c/\sum_c}\right]^{-I_s/\gamma} - \mathrm{e}^{-\tau_h/\sum_h}\right\}^2} = 0 \tag{11}$$

where $\varphi = T_c/T_{c0}$ $(\varphi > 1)$, therefore the COP can be rewritten as

$$\varepsilon = \frac{(\varphi - 1)\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)}{\gamma \frac{\varepsilon_C + 1}{\varepsilon_C} \varphi \frac{1 - \left[\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)\varphi + \mathrm{e}^{-\tau_c/\sum_c}\right]^{-I_s/\gamma}}{\left[\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)\varphi + \mathrm{e}^{-\tau_c/\sum_c}\right]^{-I_s/\gamma} - \mathrm{e}^{-\tau_h/\sum_h}} \left(1 - \mathrm{e}^{-\tau_h/\sum_h}\right) - (\varphi - 1)\left(1 - \mathrm{e}^{-\tau_c/\sum_c}\right)}. \tag{12}$$

Generally, the COP under the maximum Ω criterion can be derived by combining Eqs. (11) and (12), which will be discussed in the following parts.

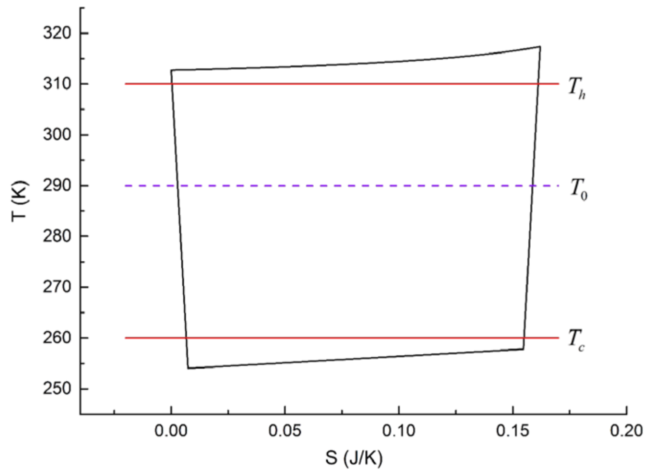

Fig. 2. The T–S diagram of an optimal refrigerator cycle where $I_s = \gamma = 1.1$, $T_h = 310$ K, $T_c = 260$ K, $T_0 = 290$ K, $\tau_h/\Psi_h = \tau_c/\Psi_c$ and $c_c m = 10$ J/K.

3. Optimal COP under the conditions where $I_s = \gamma$

Under the situations where $I_s/\gamma = 1$, as $I_s \geq 1$, therefore $\gamma \geq 1$. Eqs. (11) and (12) can be rewritten as

$$\frac{\left[\left(1-e^{-\tau_h/\sum_h}\right)\varphi\right]^2}{\left\{1-e^{-\tau_h/\sum_h}\left[\left(1-e^{-\tau_c/\sum_c}\right)\varphi+e^{-\tau_c/\sum_c}\right]\right\}^2} = \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s} \tag{13}$$

and

$$\varepsilon = \frac{1}{\gamma\frac{\varepsilon_C+1}{\varepsilon_C}\frac{\left(1-e^{-\tau_h/\sum_h}\right)\varphi}{1-e^{-\tau_h/\sum_h}\left[\left(1-e^{-\tau_c/\sum_c}\right)\varphi+e^{-\tau_c/\sum_c}\right]}-1}. \tag{14}$$

The solution of Eq. (13) gives the optimal φ, then substitute it into Eq. (14), thus

$$\varepsilon_E{}^* = \frac{\varepsilon_C}{(\varepsilon_C+1)\sqrt{I_s(T_c/T_0+\varepsilon_C)/\varepsilon_C}-\varepsilon_C}. \tag{15}$$

Eq. (15) gives the upper bound of the COP at the maximum ecological criterion which is independent of the time duration in each process and the heat conductance, The T–S diagram of an optimal cycle is shown in Fig. 2. The heat absorbing and releasing processes are not isothermal. And the non-isentropic processes are also presented. When the internal dissipations are vanished, thus $I_s = \gamma = 1$, and Eq. (15) is reduced to

$$\varepsilon_{E,endo}{}^* = \frac{\varepsilon_C}{(\varepsilon_C+1)\sqrt{(T_c/T_0+\varepsilon_C)/\varepsilon_C}-\varepsilon_C}. \tag{16}$$

It is the same as that obtained through the endoreversible Carnot model [19]. However they have different physical meanings and the optimization spaces are different. In the endoreversible Carnot model, the COP under the maximizing E criterion is obtained by with respect to the time durations of the heat absorbing and releasing processes, while in this model, it is obtained by maximizing the E function with respect to the initial temperature of the working medium, and the time durations are treated as constants. Unlike the CA refrigerators, in this model the temperature of the working medium in either heat exchanging process does not keep constant. Therefore it should be more practical and realistic than the endoreversible Carnot model.

4. Optimal COPs under the conditions where $I_s \neq \gamma$

We define $\tau/\sum$ as the dimensionless contact time, connoting the equilibrium degree of the temperature between the working medium and heat reservoir. Larger $\tau/\sum$ means the working medium contacts longer with the heat reservoirs, and will lead to higher final temperature in the heat absorbing process and lower one in the heat releasing process. Under the conditions where $I_s/\gamma \neq 1$, Eq. (11) is transcendental and cannot be solved explicitly. First, we analyze the optimal COPs under two special cases with dimensionless contact time limits ($t/\sum \to 0$, and $t/\sum \to \infty$). Then numerical calculations are conducted to investigate the general impacts of the parameters ($I_s, \gamma, \tau/\sum$) on the optimal COPs.

4.1. Short contact time limits

Under the conditions where $t/\sum \to 0$, the heat absorbing and releasing processes are both so short that the final temperature of the working substance is almost equal to its initial temperature after either process. We expand $\exp(-t/\sum)$ to the first order of $t/\sum$, Eqs. (11) and (12), can be reduced as

$$\left[\frac{\varphi}{1-\frac{I_s}{\gamma}\frac{\tau_c/\sum_c(\varphi-1)}{\tau_h/\sum_h}}\right]^2=\frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s} \tag{17}$$

and

$$\varepsilon=\frac{1}{I_s\frac{\varepsilon_C+1}{\varepsilon_C}\frac{\varphi}{1-\frac{I_s}{\gamma}\frac{\tau_c/\sum_c(\varphi-1)}{\tau_h/\sum_h}}-1}. \tag{18}$$

Combining Eqs. (17) and (18), we have the same expression as Eq. (15). It is independent of the heat capacity ratio and the heat conductance. Under the short contact time limit, the heat absorbing and releasing processes are nearly isothermal. The heat capacities have no impact on the temperature changes during the heat exchanging processes. Therefore under this situations this model returns to the irreversible Carnot refrigerator with internal dissipations.

4.2. Long contact time limits

Under the conditions where $t/\sum \to \infty$, the contact time is long enough that heat exchange between the working substance and heat reservoirs is sufficient and the final temperature of the working substance is almost equal to that of the heat reservoir. The exponential terms $\exp(-t/\sum)$ can be eliminated, therefore Eqs. (11) and (12) are simplified as

$$\varphi^{I_s/\gamma+1}=\frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s} \tag{19}$$

and

$$\varepsilon=\frac{\varphi-1}{\gamma\frac{\varepsilon_C+1}{\varepsilon_C}(\varphi^{I_s/\gamma+1}-\varphi)-\varphi+1}. \tag{20}$$

The solution of Eq. (19) gives the optimal φ, then substitute it into Eq. (20), we have

$$\varepsilon_E=\frac{[(T_c+\varepsilon_C T_0)/\varepsilon_C T_0 I_s]^{\gamma/(I_s+\gamma)}-1}{\gamma\frac{\varepsilon_C+1}{\varepsilon_C}(T_c+\varepsilon_C T_0)/\varepsilon_C T_0 I_s-\left(\gamma\frac{\varepsilon_C+1}{\varepsilon_C}+1\right)[(T_c+\varepsilon_C T_0)/\varepsilon_C T_0 I_s]^{\gamma/(I_s+\gamma)}+1}. \tag{21}$$

Eq. (21) depends on the heat capacity ratio and the internal dissipation parameter.

4.3. General conditions with finite contact times

Under the situations with finite dimensionless contact times, numerical calculations are conducted to investigate the impacts of the parameters on the optimal COPs under the ecological criterion. As depicted in Figs. 3 and 4, when the dimensionless contact times are prescribed, the optimal COP decreases with the increasing internal dissipation parameter, while it increases with increasing heat capacity ratios. Besides, when I_s is fixed, the optimal COP achieves its upper and lower bounds when $\gamma \to \infty$ and $\gamma \to 0$.

As shown in Fig. 5, when $\gamma > I_s$ and $\tau_h/\sum_h$ is fixed, the optimal COP increases with increasing $\tau_c/\sum_c$ in a certain interval, and achieves its lower and upper bounds under the asymmetric limits: $\tau_c/\sum_c \to 0$ and $\tau_c/\sum_c \to \infty$, respectively. The lower bound is $\varepsilon_E{}^*$. When $\gamma < I_s$, the optimal COP decreases with increasing $\tau_c/\sum_c$ in a certain interval and achieves its lower and upper bounds under the asymmetric limits: $\tau_c/\sum_c \to \infty$ and $\tau_c/\sum_c \to 0$. The upper bound is $\varepsilon_E{}^*$. As mentioned before when $\gamma < I_s$, the optimal COP is $\varepsilon_E{}^*$ and is independent of $\tau_c/\sum_c$.

As we can see in Fig. 6, when $\gamma > I_s$ and $\tau_c/\sum_c$ is fixed, the optimal COP increases with increasing $\tau_h/\sum_h$ in a certain interval, and achieves its lower and upper bounds when $\tau_h/\sum_h \to 0$ and $\tau_h/\sum_h \to \infty$, respectively. The lower bound is $\varepsilon_E{}^*$. When $\gamma < I_s$, the optimal COP decreases with increasing $\tau_h/\sum_h$ in a certain interval and achieves its lower and upper bounds when $\tau_h/\sum_h \to \infty$ and $\tau_h/\sum_h \to 0$. The upper bound is $\varepsilon_E{}^*$. As mentioned before $\gamma < I_s$, the optimal COP stays constant and is equal to $\varepsilon_E{}^*$.

According to the above analysis, when $\gamma > I_s$, the optimal COP will increase with increasing $\tau/\sum$ will achieve its maximum value when $\tau/\sum \to \infty$. The lower bound is achieved when $\tau/\Psi \to 0$, and is equal to $\varepsilon_E{}^*$, and is independent of the heat capacity ratio. When $\gamma < I_s$, the optimal COP will decrease with increasing $\tau/\sum$, will obtain its minimum value when $\tau/\sum \to \infty$. The upper bound is achieved when $\tau/\sum \to 0$, and is equal to $\varepsilon_E{}^*$ and is independent of the heat

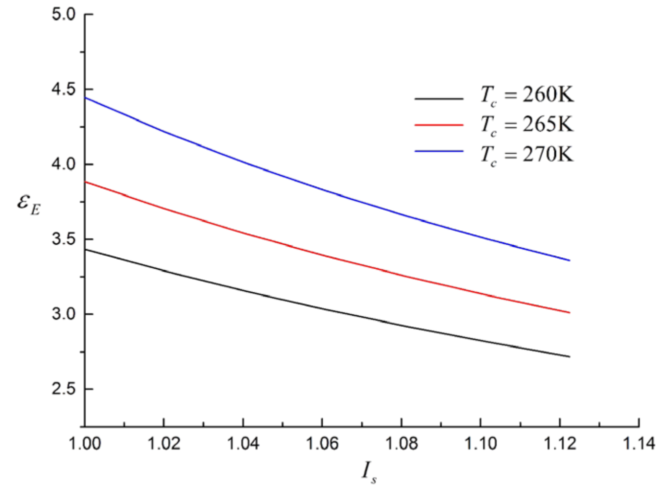

Fig. 3. Optimal COPs with I_s under different temperatures of the cold reservoir ($T_c = 260$ K, 265 K, 270 K), where, $T_h = 310$ K, $T_0 = 290$ K, $\gamma = 0.8$ and $\tau_c/\sum_c = \tau_h/\sum_h = 1$.

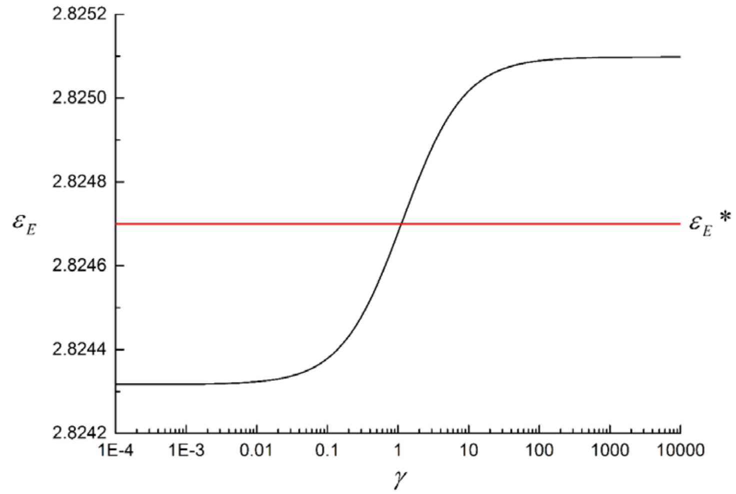

Fig. 4. Optimal COPs with different heat capacity ratios where $T_h = 310$ K, $T_c = 260$ K, $T_0 = 290$ K, $I_s = 1.1$ and $\tau_c/\sum_c = \tau_h/\sum_h = 1$.

capacity ratio. Therefore the general lower and upper bounds of the optimal COP can be obtained in the situations where $\tau/\sum \to \infty$, by applying the asymmetric heat capacity limits: $\gamma \to 0$ and $\gamma \to \infty$, respectively. According to Eq. (21) with respect to the asymmetric heat capacity ratio limits, the lower and upper bounds of the COP of the general refrigerators under the maximum ecological criterion are given by

$$\varepsilon_E^- = \frac{\varepsilon_C \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s}}{(\varepsilon_C + 1)\frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0} - I_s(\varepsilon_C + 1) - \varepsilon_C \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s}} \tag{22}$$

and

$$\varepsilon_E^+ = \frac{\varepsilon_C \left(\frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s} - 1\right)}{\varepsilon_C \left(1 - \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s}\right) + (\varepsilon_C + 1)\frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0} \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0 I_s}}. \tag{23}$$

5. General endoreversible refrigerators

Without considering the internal dissipations, the general refrigerator model is endoreversible, that is to say the irreversibility is only located in the heat exchanging processes. In this sense, $I_s = 1$ and $\varepsilon_E{}^*$ is recovered to $\varepsilon_{E,endo}{}^*$. According

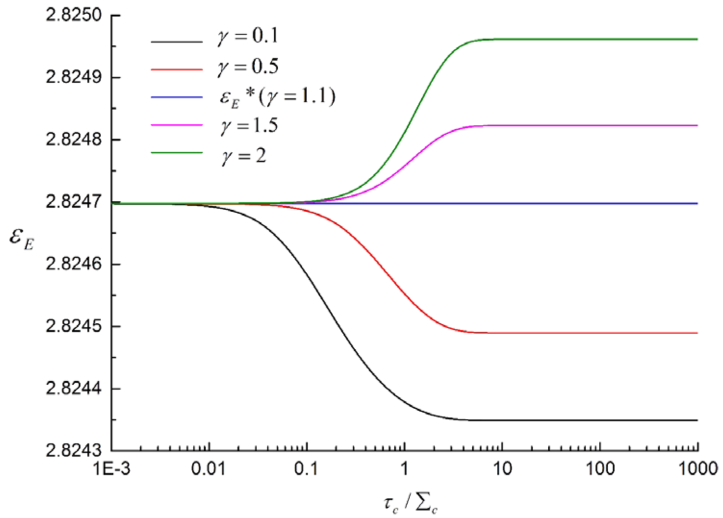

Fig. 5. Optimal COPs with dimensionless contact times of the heat releasing process under different heat capacity ratios ($\gamma = 0.1, 0.5, I_s, 1.5, 2$), where $T_h = 310$ K, $T_c = 260$ K, $T_0 = 290$ K, $I_s = 1.1$ and $\tau_h/\sum_h = 1$.

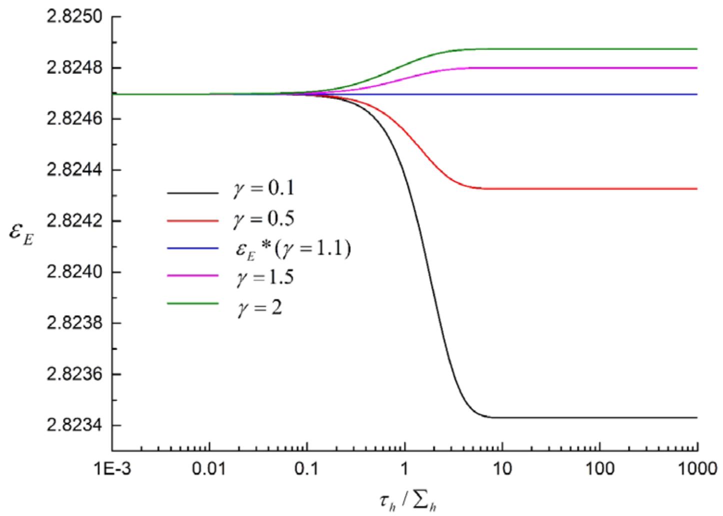

Fig. 6. Optimal COPs with dimensionless contact times of the heat absorbing process under different heat capacity ratios ($\gamma = 0.1, 0.5, I_s, 1.5, 2$), where $T_h = 310$ K, $T_c = 260$ K, $T_0 = 290$ K, $I_s = 1.1$ and $\tau_c/\sum_c = 1$.

to the aforementioned analysis, when $\gamma > 1$, the lower bound of the COP at the maximum ecological criterion is $\varepsilon_{E,endo}{}^*$, When $\gamma < 1$, the upper bound is $\varepsilon_{E,endo}{}^*$. When $\gamma = 1$, the optimal COP is equal to $\varepsilon_{E,endo}{}^*$.

Therefore in the endoreversible refrigerator cycles such as reversed Diesel cycle ($c_h = c_p, c_c = c_v$), reversed Brayton cycle ($c_c = c_h = c_p$), and reversed Otto cycle ($c_c = c_h = c_v$), where the heat capacity in the heat absorbing process is not less than that in the heat releasing process, the lower bound of the COP under the ecological criterion is $\varepsilon_{E,endo}{}^*$, while in the cycles such as reversed Atkinson cycle ($c_h = c_v, c_c = c_p$) where the heat capacity in the heat absorbing process is less than that in the heat releasing process, $\varepsilon_{E,endo}{}^*$ is the upper bound. This might be of great guidance for designing and operating actual refrigerators.

Furthermore, by substituting $I_s = 1$ to Eqs. (22) and (23), the general lower and upper bounds of the COP for the general endoreversible refrigerator are given by

$$\varepsilon_{E,endo}{}^- = \frac{\varepsilon_C \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0}}{(\varepsilon_C + 1)\frac{T_c}{\varepsilon_C T_0} - \varepsilon_C \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0}} \tag{24}$$

$$\varepsilon_{E,endo}{}^+ = \frac{\varepsilon_C}{(\varepsilon_C + 1)\frac{T_c+\varepsilon_C T_0}{T_c} \ln \frac{T_c+\varepsilon_C T_0}{\varepsilon_C T_0} - \varepsilon_C}. \tag{25}$$

6. Conclusions

In conclusion, we have conducted an analysis of COP and its bounds at maximum ecological criterion for general refrigerators. For generality, both the non-isothermal heat transfer processes and the internal dissipations are considered. Under the situations where $I_s = \gamma$, the optimal COP is $\varepsilon_E{}^*$, and is independent of the time duration completing each process and the heat conductance. Under the conditions where $I_s \neq \gamma$, first, we analyze the optimal COPs under two special cases with dimensionless contact time limits ($\tau/\sum \to 0$, and $\tau/\sum \to \infty$). Then numerical calculations are conducted to investigate the general impacts of the parameters on the optimal COPs under the ecological criterion. The optimal COP decreases with the increasing internal dissipation parameter, while it increases with increasing heat capacity ratios. We further proved that in the long contact time limits, the general lower and upper bounds of the optimal COP can be, respectively, obtained by applying the asymmetric heat capacity limits: $\gamma \to 0$ and $\gamma \to \infty$. Finally the general upper and lower bounds of the optimal COP are proposed.

Furthermore under maximum ecological criterion, the COP of general endoreversible refrigerators have also been studied. And the COP bound of different kinds of refrigerators have been analyzed. In the endoreversible refrigerator cycles such as reversed Diesel cycle, reversed Brayton cycle, and reversed Otto cycle, where the heat capacity in the heat absorbing process is not less than that in the heat releasing process, the lower bound of the COP under the ecological criterion is $\varepsilon_{E,endo}{}^*$, while in the cycles such as reversed Atkinson cycle where the heat capacity in the heat absorbing process is less than that in the heat releasing process, $\varepsilon_{E,endo}{}^*$ is the upper bound. In addition, the general upper and lower bounds of the COP for general endoreversible refrigerators have been also obtained.

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