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Fluctuation and inertia

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Abstract

How the principle of inertia survives quantum fluctuations is an interesting question. Smolin has proposed a hypothesis that quantum fluctuations are in fact real statistical fluctuations. In this work, combining the works on Hawking-Unruh radiation and Jacobson's idea in his thermodynamics derivation of Einstein equation, we confirmed Smolin's guess: the quantum fluctuations leading to Hawking-Unruh radiation, satisfying the fluctuation theorem, are statistical fluctuations. Therefore, inertia is found to be a result of the second law of thermodynamics: the principle of entropy increases has the tendency to eliminate the effects of fluctuations and makes accelerated observers express inertia force.

1. Introduction

There should be no doubt that the principle of inertia can be among the most important laws of physics. Look backing at the history of physics, we can find that the ever-deepening understanding of inertia has played important roles in both the development of classic mechanics and the

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discovery of general relativity. Study on the details of the gravitational coupling between matter and spacetime which results in inertia may provide us more useful information.

According to the uncertainty principle, the energy of a particle has big fluctuations during short time intervals because of quantum fluctuations. However, the principle of inertia can survive these quantum fluctuations when the time interval is long enough. This naturally reminds us of the relationship between the second law of thermodynamics and statistical fluctuation. In fact, Smolin has proposed a hypothesis that quantum fluctuations are in fact real statistical fluctuations in 1986 [1]. A series of important theoretical advances have enabled us to revisit this problem in more depth. The discovery of fluctuation theorem [2,3] gives an analytical expression for the probability of fluctuations quantitatively. The close relationships between the second law of thermodynamics and some basic assumptions in both relativity and quantum mechanics were discovered [4–7]. The attempt to interpret the gravity and inertia in a thermodynamic context has been one important topic of the theoretical progress in gravitational interaction [8–11]. In [8], Jacobson deduced the Einstein equation using thermodynamics method by assuming that "the fundamental thermodynamic relation $\delta Q = TdS$ hold for all the local Rindler causal horizons through each spacetime point". Recently, a quantum picture of space as Bose-Einstein condensates of gravitons has been developed [12].

The plan of the paper is as follows. In the second section, using the works about the thermal spectrum of Hawking-Unruh radiation [13–17] and Jacobson's idea in his thermodynamics derivation of Einstein equation [8], we confirmed Smolin's guess [1]: quantum fluctuations, satisfying the fluctuation theorem, are real statistical fluctuations. In the third section, we analyzed the energy exchange between accelerated objects and space during the acceleration process from the perspective of different observers.

2. Quantum fluctuations as statistical fluctuations

According to the fluctuation theorem [2,3], the probabilities of a fluctuation with an entropy decrease of ΔS and its reverse process with an entropy increase of ΔS satisfies the following equation

$$\frac{p(-\Delta S)}{p(+\Delta S)} = \exp(-\Delta S) \tag{1}$$

If the quantum fluctuations of vacuum resulting in Hawking-Unruh radiation are statistical fluctuations, they should satisfy fluctuation theorem described by equation (1).

Our analysis will be based on Sannan's work [15]. Combining Damour and Ruffini's approach [14] with the intuitive method of scattering amplitudes due to Feynman [18], he obtained the probability distributions of both bosons and fermions emitted from the black horizon to infinity [15].

2.1. Quantum fluctuations and Hawking-Unruh radiation

According to Hawking [13], a black hole, which can be regarded as an excited state of the gravitational field, can decay quantum mechanically because of quantum fluctuation of the metric: energy tunnels out of the potential well of a black hole and gives rise to the creation of a pair: one particle (positive energy) going out and one antiparticle (negative energy) falling back toward the singularity. Using a method of barrier penetration in curved spaces, Damour and Ruffini [14] obtained the relative scattering probability of the outgoing wave by the horizon as

$$p_\omega = \exp(-8\pi M\omega) \tag{2}$$

where M is the mass of a Schwarzschild black hole and ω is the energy mode of the particle or the antiparticle.

According to Feynman [18], the probability p_ω is also the relative probability of creating a particle-antiparticle pair just outside the horizon

$$p_\omega = \frac{p_+}{p_-} \tag{3}$$

where p_+ is the probability of creating a particle-antiparticle pair from vacuum caused by quantum fluctuation, while p_- is the probability that the final state is a vacuum, which can be seen as a re-annihilation of the creating particle-antiparticle pair. Combining equation (2) with equation (3), Sannan [15] obtained the probability distributions of both bosons and fermions emitted from the black horizon to infinity.

For an observer outside the horizon, black hole loses energy, accompanied by a decrease in the area of its horizon when he/she receives a particle of hawking radiation. According to the work of Bekenstein [19], this is an entropy reduction process for the black hole. Black hole recovers the energy it loses when the re-annihilation of the creating particle-antiparticle pair happens. After recovering the energy, the horizon area increases, which means that it is a spontaneous entropy increase process. The temperature of a Schwarzschild black hole with a mass of M is

$$T_g = \frac{1}{8\pi M} \tag{4}$$

The entropy change of the horizon caused by the quantum fluctuation resulting in the radiation of a particle with energy of ω is

$$\Delta S = -\frac{\omega}{T_g} \tag{5}$$

Substituting equation (5) into (2) gives

$$p_\omega = \exp(-\Delta S) \tag{6}$$

Substituting equation (6) into (3) gives

$$\frac{p_+}{p_-} = \exp(-\Delta S) \tag{7}$$

Comparing equation (7) with equation (1), we can found that the quantum fluctuations resulting in Hawking radiation satisfy fluctuation theorem. In [16], Zhao and Gui have proved that Damour-Ruffini's scheme for Hawking radiation [14] and Unruh's scheme [20] dealing with the Hawking-Unruh effect are equivalent. Using High-dimensional global embedding Minkowski spacetimes, Deser and Levin [17] also discussed the equivalence of Hawking and Unruh temperatures. Therefore, the above discussion of Hawking radiation should also apply to Unruh radiation. In this way, the quantum fluctuations of vacuum resulting in Hawking-Unruh radiation satisfy fluctuation theorem and are real statistical fluctuations as guessed by Smolin [1]. The analysis of Bell and Leinaas [21] about the interactions between accelerated electrons and Unruh radiation provides evidence to support this conclusion.

2.2. Interactions between accelerated electrons and Unruh radiation

According to the work of Gibbons and Hawking [22], the radiation of horizon is dependent upon the measurements of the observer: if the observer chooses not to absorb any radiation, there is no change in area of the horizon. In the case discussed by Bell and Leinaas [21], the accelerated electron is the observer of Unruh radiation. When it absorbs a particle with energy of ΔE from the horizon, its energy increases and an up transition from the ground spin state (E_0) to the excited spin state ($E = E_0 + \Delta E$) occurs. At the same time, the horizon loses energy, accompanied by a reduction in area (and its entropy). During a reverse down transition from the excited spin state to ground spin state, the particles return energy to the horizon, the area (and entropy) of which increases.

Bell and Leinaas [21] found that the up and down transition probabilities between the ground spin state (E_0) and the excited spin state ($E = E_0 + \Delta E$) of a uniformly accelerating electron with an acceleration of a satisfy

$$\frac{P_+}{P_-} = \exp(-\frac{2\pi \Delta E}{a}) \tag{8}$$

where a is the acceleration of the accelerated electron, P_+ and P_- are the up and down probabilities, respectively. The Unruh temperature registered by the accelerated electron [20] is

$$T_U = \frac{a}{2\pi} \tag{9}$$

Substituting equation (9) into (8), we can get

$$\frac{P_+}{P_-} = \exp(-\frac{\Delta E}{T_U}) \tag{10}$$

When Jacobson [8] deduced the Einstein equation using thermodynamics method, the fundamental thermodynamic relation

$$\delta Q = T dS \tag{11}$$

is assumed to hold for all the local Rindler causal horizons through each spacetime point, with δQ and T are the energy flux and the Unruh temperature observed by an accelerated observer just inside the horizon, respectively. In other word, the energy flux across a causal horizon is a kind of heat flow in spacetime dynamics. For an isothermal process, integrating equation (11) gives

$$\Delta Q = T \Delta S \tag{12}$$

The transitions between the two spin states are caused by the radiation field with temperature of Unruh temperature. In other word, the excited electron harvests energy form space through the horizon to make up the energy difference of ΔE between the ground state and the excited state. Therefore, the energy flux across the causal horizon, ΔQ, during the up transition of the electron is

$$\Delta Q = -\Delta E \tag{13}$$

The negative sign indicates that space loses energy. From equations (12) and (13), the change in the entropy of horizon, ΔS, can be written as

$$\Delta S = \frac{\Delta Q}{T_U} = \frac{-\Delta E}{T_U} \tag{14}$$

Substituting equation (14) into equation (10) gives

$$\frac{P_+}{P_-} = \exp(\Delta S) \tag{15}$$

3. Descriptions of an acceleration process from different perspectives

The discussion in the above section can help us understand Verlinde's hypothesis that inertia force is result of entropy increase from a deeper level [10].

3.1. Verlinde's entropic inertia force

Motivated by Bekenstein's original thought experiment leading to black hole entropy [23], Verlinde [10] assumed that when a particle of mass m changes its position by Δx with respect to some holographic screen by one Compton wavelength, the change of entropy associated with the information on the holographic screen equals

$$\Delta S = 2\pi \quad \text{when} \quad \Delta l = \frac{1}{m} \tag{16}$$

and inertia force is an entropic force equals to

$$F_i \Delta l = T_U \Delta S \tag{17}$$

where T is the Unruh temperature experienced by the accelerated particle as described by equation (9), he derived Newton's law of inertia:

$$F_i = ma \tag{18}$$

where F_i is in fact the modulus of the inertia force (there will be a negative sign to the right of the equation (18) if the directions of inertia force and acceleration are considered).

As mentioned above, the radiation of horizon is dependent upon the measurements of the observer [22]. An accelerated observer, Alice, can obtain the Unruh temperature of the horizon by analyzing the energy of the particles of Unruh radiation she receives from the horizon. The instantaneously co-moving frame of Alice is a non-inertial frame, which means that she won't realize that she is accelerating and therefore thinks her energy is constant. In this way, Alice will return the same energy to the horizon. Therefore, from the view of Alice, she thermalizes at the Unruh temperature and remains in equilibrium with the Rindler horizon by exchanging particle with it but there is no net radiation of energy between them, this agrees with the work of Ford and O'Connell [24].

The energy recovery of the horizon is a process of increasing entropy, which makes Alice express an inertia force.

$$F_i \Delta l = \Delta Q = T_U \Delta S \tag{19}$$

where ΔQ is the energy recovered by the horizon from Alice. The inertia force actually acts on Alice's local space.

3.2. Bekenstein's generalized second law of thermodynamics

According to Verlinde [10], Alice merges with the microscopic degrees of freedom on the horizon and shares its temperature. Considering the conservation of energy, Alice will lose the

same amount of energy when she return the energy of ΔQ to the horizon: it is an isothermal heat transfer process between Alice and the horizon. The change of Alice's entropy is

$$\Delta S_A = -\frac{\Delta Q}{T_U} \tag{20}$$

For a composite system consisting of Alice and the horizon, the total entropy change of this process will be

$$\Delta S_{total} = \Delta S + \Delta S_A = 0 \tag{21}$$

The above analysis can also apply to the process during which Alice absorbs Unruh radiation from the horizon. In this way, Bekenstein's generalized second law of thermodynamics for black hole horizons [19] can still be applied to the Rindler horizon of an accelerated observer.

For an inertial observer Bob, the description of the acceleration of Alice will be different. From the view of Bob, there is no inertia force acting on the Alice: the work done by external force is equal to the increase of kinetic energy and the heat dissipation caused by irreversible loss,

$$F\Delta l = T_1 \Delta S_1 + \Delta E_k \tag{22}$$

where T_1 is the common temperature of Alice observed by Bob, ΔS_1 is the entropy increase because of the heat dissipation caused by irreversible loss, ΔE_k is the change in the kinetic energy of Alice. For an ideal reversible process without any heat dissipation, $\Delta S_1 = 0$. Unruh temperature is in fact the temperature of vacuum observed by Alice where Bob (as an inertial observer) would observe none (the temperature of vacuum is the zero in Bob's thermometric scale). Therefore T_1 is an temperature difference between the vacuum and Alice from the view of Bob.

In summary, an ideal reversible process of acceleration is an isentropic process form the view of an inertial observer or a non-inertial observer. For the problem of gravity, Rindler horizons and their Unruh temperatures will be replaced by holographic screens and their generalized Bekenstein-Hawking temperature [25], respectively. In this way, the above analysis of inertial forces should also apply to gravity. Therefore, the total entropy increase of an ideal acceleration caused by gravity will be 0, which agrees with the reversible nature of gravity as a conservative force [26].

4. Conclusion and discussion

In this work, first, we confirmed Smolin's guess: the quantum fluctuations leading to Hawking-Unruh radiation, satisfying the fluctuation theorem, are real statistical fluctuations [1]. Therefore, inertia is found to be a result of the second law of thermodynamics: the principle of entropy increases has the tendency to eliminate the effects of fluctuations and makes accelerated observers express inertia force. Then, we analyzed the energy exchange between accelerated objects and space during the acceleration process from the perspective of different observers. It is found that the total entropy change of an ideal reversible acceleration is 0 form the view of an inertial observer or a non-inertial observer.

In this work, we assumed that the heat transfer between the particles and the horizon is an isothermal reversible process. This assumption is also adopted by related works [8,10]. However, ideal reversibility means that the driving force is zero and the process takes in infinite time to complete. An actual physical process is more or less irreversible accompanied by an additional

entropy increase. The Larmor radiation of a charged particle may be related to this additional entropy increase.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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