A study on the multi-field synergy principle of convective heat and mass transfer enhancement

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A B S T R A C T
In this study, a constitutive relation between fluid power flux and pressure gradient was discussed, and a conservation equation of fluid mechanical energy was introduced to describe convectional phenomena that is essential to convective heat and mass transfer. The multi-field synergy relations among velocity, pressure, temperature and component concentration were revealed according to the synergy equations based on the conservations of thermal energy, mechanical energy, component mass and fluid momentum to analyze physical characteristic of enhancing convective heat and mass transfer. A new experimental correlation of \( \text{Eu} \) number versus \( \text{Re} \) and \( \text{Li} \) numbers was obtained for the first time through theoretical derivation and experiment to test hydrodynamic performance of heat transfer tube. A circular tube inserted by longitudinal swirl flow generator with center-connected rectangle rods was investigated through numerical simulation and PIV experiment to validate the synergy relation in convective heat transfer enhancement.

1. Introduction

Since the last century, people had been paying much attention to explore the physical mechanism of convective heat and mass transfer based on the conservation laws that were expressed by the primary conservation equations of mass, momentum, and energy [1–5]. However, when people want to solve for velocity and pressure in pure convection, they find no pressure equation to couple with the Navier-Stokes equation that can be disposed as a discrete equation for velocity. So they have to use continuity equation to satisfy mass conservation in the solution. This means that the conservation equations in fluid dynamics are not closed. Therefore, in the SIMPLE algorithm proposed by Patankar and Spalding in 1972 [6], when solving the Navier-Stokes equation, a set of algebra equations of correcting pressure should be solved first, which is derived by satisfying continuity equation to correct the velocity and pressure, then the velocity and pressure are found through the iteration method. Since 1972, the SIMPLE and its family algorithms had been widely used to solve the Navier-Stokes equation [7–10]. Furthermore, in the simulation of convective heat and mass transfer, people used the SIMPLE algorithm by coupling the Navier-Stokes equation with the conservation equations of thermal energy and component mass to find multi-physical fields of velocity, pressure, temperature and concentration [11–18]. In our previous study, however, a mechanical energy conservation equation in terms of pressure was established by constructing a constitutive relation between fluid power flux and pressure gradient. Thus, the Navier-Stokes equation could be solved by directly coupling the proposed conservation equation to find velocity and pressure of the fluid [19]. This may provide an alternative method for simulating pure convection or convective heat and/or mass transfer.

In the study of convective heat transfer enhancement, Guo et al. provided a new insight into the physical mechanism of convective heat transfer, which depends on the synergetic relation between velocity and temperature fields. They proposed a principle of field synergy through establishing a synergy equation based on thermal energy conservation, which demonstrates the relation between the Nusselt number and the dot product of velocity vector and temperature gradient. Under the same boundary conditions of velocity and temperature, the better the synergy between velocity and temperature fields is, the higher the heat transfer intensity will be [20–23]. Tao et al. proved the field synergy principle by various numerical simulations for the laminar and turbulent flows. They also verified that the field synergy principle could be used as a guide to design heat transfer units and heat exchangers [24–26]. To further develop this principle, Liu et al. extended the synergy relations among variables from two physical fields involving velocity and temperature to three physical fields including velocity, temperature and pressure for both laminar and turbulent flows [27,28]. In this work, we aim to explore a general synergy principle by...
Introducing a set of synergy equations based on the conservation of thermal energy, mechanical energy, component mass and fluid momentum to reveal the physical mechanism of convective heat and mass transfer [29]. For this purpose, the conservation equation in terms of pressure was further developed as a synergy equation, from which a dimensionless experimental correlation was obtained for hydrodynamic experiments applied to the plain and heat transfer enhanced tubes.

2. Multi-field synergy principle of convective heat and mass transfer

2.1. Conservation equation of mechanical energy

As we know that the Fourier's law shows a constitutive relation between fluid heat flux and temperature gradient [30,31], which is expressed as

\[ \bar{q} = -k \nabla T, \]  

where \( \bar{q} \) represents the magnitude and direction of heat flux at any point of flow field, reflecting the ability of the fluid to transfer heat in the way of heat convection, W/m². It should be noticed that the Fourier's law here describes the mechanism of heat convection rather than heat conduction. Thus, it is necessary to define \( k \) as a heat convection coefficient of the fluid, which is different from traditional definition in the textbooks.

In addition, a constitutive relation between fluid power flux and pressure gradient was established to reflect the inherent nature of the fluid [19],

\[ \bar{\omega} = -\zeta \nabla p, \]  

where \( \bar{\omega} \) represents the magnitude and direction of power flux at any point of flow field, reflecting the ability of the fluid to do work, W/m². In Eq. (2), minus sign indicates that the direction of power flux is opposite to that of pressure gradient, and \( \zeta \) is power factor of the fluid or power consumption coefficient, m²/s. The pump or fan is used to provide a driving force to the fluid, which is the source for the distribution of power flux in a flow field. Then a mechanical energy conservation equation in the steady state for the incompressible fluid was derived as [19],

\[ \mathbf{U} \cdot \nabla p = \zeta \nabla^2 p, \]  

or

\[ \rho \mathbf{U} \cdot \nabla p = \zeta \nabla^2 p, \]  

where \( \zeta \) stands for power factor of the fluid not including fluid density, kg/(m s). It is worth to mention that Eq. (3) can be treated as a basic governing equation for pure convection or convective heat and/or mass transfer.

As shown in Fig. 1, we drew a diagram of treble circles to demonstrate the mutual relations of convective heat and mass transfer. The upper circle represents pure convection illustrated by conservation equations of fluid momentum and mechanical energy for velocity and pressure. The bottom left circle indicates convective heat transfer illustrated by conservation equations of fluid momentum, mechanical energy and thermal energy for velocity, pressure, and temperature. The bottom right circle denotes convective mass transfer illustrated by conservation equations of fluid momentum, mechanical energy and diffusive mass for velocity, pressure and component concentration. In whatever circle, the conservation equations are closed now for velocity, pressure, tem-
perature or component concentration to simulate pure convection or convective heat and/or mass transfer. In addition, continuity equation, i.e., \( \nabla \cdot \mathbf{U} = 0 \), should be satisfied in all circles.

2.2. Synergy equations of convective heat and mass transfer

Integrating the term of right hand side in Eq. (3) in a two-dimensional parallel tunnel with length \( L \) and height \( l \), we have

\[
\int_0^L \int_0^l (\mathbf{U} \cdot \nabla p) \, dx \, dy = \int_0^L \int_0^l \nabla^2 p \, dx \, dy, \quad (5)
\]

For a constant \( \zeta \), the term of right hand side in Eq. (5) becomes

\[
\int_0^L \int_0^l \nabla^2 p \, dx \, dy = \zeta \Delta p, \quad (6)
\]

where \( \Delta p \) is pressure difference between the inlet and outlet cross-sections. It should be noted that the pressure on any cross-section in the tunnel is variable, which can only be approximated to be constant in a simplified analysis. The term of left hand side in Eq. (3) can be expressed as

\[
\int_0^L \int_0^l (\mathbf{U} \cdot \nabla p) \, dx \, dy = \int_0^L \int_0^l \left\{ (\mathbf{U} \cdot \nabla p) \left| u_m \right| (\rho u_m^2) \right\} \, dx \, dy, \quad (7)
\]

where \( u_m \) is average velocity of the fluid, and dimensionless numbers are

\[
X = \frac{x}{L}, \quad Y = \frac{y}{l}, \quad \mathbf{U} = \frac{\mathbf{U}}{u_m}, \quad \nabla p = \frac{\nabla p}{\rho u_m^2}. \quad (8)
\]

Substituting Eqs. (6) and (7) into Eq. (5) yields

\[
\frac{\Delta p}{\rho u_m^2} = \frac{u_m}{v} \int_0^l \int_0^l (\mathbf{U} \cdot \nabla p) \, dx \, dy. \quad (9)
\]

Then, we can have a synergy equations based on fluid mechanical energy conservation,

\[
Eu = \text{ReLi} \int_0^L \int_0^l (\mathbf{U} \cdot \nabla p) \, dx \, dy, \quad (10)
\]
or

\[
Eu = \text{ReLi} \int_0^L \int_0^l -[\mathbf{U} \cdot (\nabla p)] \, dx \, dy, \quad (11)
\]

where minus sign before \( \nabla p \) represents the opposite direction of positive pressure gradient. \( Eu \) is Euler number, which reflects the relative relationship between pressure drop through the fluid and dynamic head of the fluid. \( Re \) is Reynolds number, and \( Li \) is a new dimensionless number that is defined as

\[
Li = \frac{\zeta}{v}, \quad (12)
\]

where \( Li \) number is equal to the ratio of viscous diffusion coefficient \( v \) to power consumption coefficient \( \zeta \). Obviously, \( Li \) number is equivalent to Prandtl number in convective heat transfer or Schmidt number in convective mass transfer, which reflects the relative relationship between momentum diffusion through the fluid and power consumption in the fluid. As mentioned in Refs. [20,21], the conservation and synergy equations based on thermal energy in the steady state can be described as

\[
\mathbf{U} \cdot \nabla T = a \nabla^2 T, \quad (13)
\]

\[
\text{Nu} = \text{RePr} \int_0^l \int_0^l (\mathbf{U} \cdot \nabla T) \, dx \, dy, \quad (14)
\]

where \( \text{Nu} = hL/k \) is Nusselt number, \( Pr = \nu/\alpha \) is Prandtl number, \( \alpha \) is thickness of thermal boundary layer, \( \nabla T = \nabla T/(T_w - T_f) \) is dimensionless temperature gradient, \( T_w \) is wall temperature, and \( T_f \) is fluid temperature.

Noting that the constitutive relation of \( f = -D \nabla C \) called the Fick’s law [32,33] and the dot products in Eqs. (11) and (14), the conservation and synergy equations based on diffusive mass in the steady state can be expressed as

\[
\mathbf{U} \cdot \nabla C = D \nabla^2 C, \quad (15)
\]

\[
\text{Sh} = \text{ReSc} \int_0^l \int_0^l (\mathbf{U} \cdot \nabla C) \, dx \, dy, \quad (16)
\]

where \( \text{Sh} = h_m l / D \) is Sherwood number, \( Sc = \nu / D \) is Schmidt number, \( h_m \) is thickness of concentration boundary layer, \( D \) is mass transfer coefficient, \( \nabla C = \nabla C/(C_w - C_f) \) is dimensionless concentration gradient, \( C_w \) is component concentration near the wall, and \( C_f \) is component concentration in the fluid.

The transport characteristics of the fluid can be described by the dot products in Eqs. (11), (14) and (16). The synergy relations between velocity vector \( \mathbf{U} \) and gradients \( \nabla T, \nabla C \) or \( -\nabla p \) can be expressed by the synergy angles,

\[
\beta = \arccos \frac{\mathbf{U} \cdot \nabla T}{|\mathbf{U}| |\nabla T|}, \quad (17)
\]

\[
\alpha = \arccos \frac{\mathbf{U} \cdot \nabla C}{|\mathbf{U}| |\nabla C|}, \quad (18)
\]

\[
\theta = \arccos \frac{\mathbf{U} \cdot (-\nabla p)}{|\mathbf{U}| |\nabla p|}. \quad (19)
\]

If define a synergy angle between the velocity sector \( \mathbf{U} \) and a positive pressure gradient \( \nabla p \) corresponding to the dot product in the right hand side of Eq. (10), Eq. (19) should be written as

\[
\pi - \theta = \arccos \frac{\mathbf{U} \cdot \nabla p}{|\mathbf{U}| |\nabla p|}. \quad (20)
\]

Fig. 2(a) and (b) show the three dimensional synergy relations among physical quantities at a fluid particle \( M \) in convective heat transfer and \( N \) in convective mass transfer. For a convective heat transfer process, as shown in the figure, if angle \( \beta \) is decreased, angle \( \alpha \) will be increased, which implies that more heat is transferred and more power is consumed in the same time; or vice versa. For a convective mass transfer process, similarly, if angle \( \alpha \) is decreased, angle \( \theta \) will be increased, which implies that more component mass is diffused and more power is consumed in the
same time; or vice versa. Thus, the pressure gradient $\nabla p$ is essential to both convective heat and mass transfers, corresponding to the power consumed in the transport processes. Fig. 2(a) and (b) also show that the synergy of convective heat or mass transfer is based on three physical quantity fields respectively. There exists no direct synergy relation between temperature gradient $\nabla T$ and component concentration gradient $\nabla C$.

Obviously, after a conventional CFD modeling, the further calculation based on above synergy relations can provide a quantitative evaluation to comprehensive performance of convective heat and mass transfer enhancement. In addition, the conservation and synergy equations based on fluid momentum in the steady state can be expressed as [27, 28],

$$\rho \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \mu \nabla^2 \mathbf{U},$$

where $\mathbf{U}$ is dimensionless component velocity in the $x$ direction, $\chi_1 = L_z/L$, $\chi_2 = (L - L_x)/L$ and $L_x$ stands for the length of entrance region in the two-dimensional parallel tunnel. In Eq. (21), the viscous diffusion term is related to a constitutive relation of $\tau = k \mathbf{U}$ called the Newton’s law of fluid viscosity [34].

In Eq. (22), the first and second terms on the right hand side are viscous resistances in the entrance and fully developed regions in the two-dimensional parallel tunnel. The third term on the right hand side is momentum change of the fluid represented by a dot product between velocity vector $\mathbf{U}$ and component velocity gradient $\nabla \mathbf{U}$. Thus, we can define another synergy angle,

$$\gamma = \arccos \left( \frac{\mathbf{U} \cdot \nabla \mathbf{U}}{||\mathbf{U}|| \cdot ||\nabla \mathbf{U}||} \right).$$

In Eq. (23), the direction of component velocity $u$ represents the direction of fluid mainstream in the two-dimensional parallel tunnel. The larger the angle $\gamma$, the smaller the component velocity gradient in the $x$ direction and the larger the component velocity gradient in the $y$ direction. In this case, the direction of velocity vector $\mathbf{U}$ will be more closed to the direction of fluid mainstream, which means less power will be consumed due to the change of fluid momentum. This is also true in the 3-D flow fields, which displays a mechanism of disturbing fluid [27, 28].

For a composite process of convective heat and mass transfer, the overall synergy relation of physical fields was illustrated in Fig. 3. From this figure, we can observe that the multi-field synergy in convective heat and mass transfer is demonstrated by four synergy angles between velocity vector $\mathbf{U}$ and other physical vectors in a fluid particle. However, concentration gradient can be influenced by temperature gradient, which is so-called Soret effect; and temperature gradient can be influenced by concentration gradient, which is so-called Dufour effect.

Although Eq. (22) can reflect synergy relation between velocity vector $\mathbf{U}$ and component velocity gradient $\nabla \mathbf{U}$, it is not a basic synergy equation. Therefore, we basically have three main synergy equations that are Eqs. (11), (14) and (16) and three main synergy angles expressed by Eqs. (17), (18) and (19) to demonstrate the multi-field synergy relations in convective heat and mass transfer.

2.3. Experimental correlations of convective heat and mass transfer

The dimensionless experimental correlations of convective heat transfer are usually expressed as $Nu = f(Re, Pr)$ or $Nu = aRe^n Pr^m$. If synergy angle $\beta = 0$, according to Eqs. (14) and (17), the $Nu$ number will reach a maximum value compared to other beta angles, which is the most beneficial to heat transfer. For convective mass transfer, similarly, the dimensionless experimental correlations can be expressed as $Sh = f(Re, Sc)$ or $Sh = bRe^n Sc^m$. If synergy angle $\alpha = 0$, according to Eqs. (16) and (18), the $Sh$ number will reach a maximum value compared to other alpha angles, which is the most beneficial to mass transfer.
From Eq. (10) or (11), we can have a function of $Eu = f(Re, Li)$ for the convection. If synergy angle $\theta = 0$, according to Eqs. (11) and (19), the $Eu$ number will reach a minimum value compared to other theta angles. This represents a most advantageous situation to reduce the power consumed by the fluid. Then, we can have a new dimensionless experimental correlation for the convection,
\[ Eu = cRe^{m_3}Li^{n_3}, \]  
(24)

where coefficients $c$, power exponents $m_3$ and $n_3$ are constant. At different fluid status, however, such as the laminar or turbulent flows, the values of coefficient and power exponents will be different. It is worth to note that Eq. (24), that is suitable for single-phase fluid, can't be applied to boiling heat transfer of the liquid [35].

As shown in Table 1, all constitutive, conservative and synergic equations for variables $T$, $C$ and $p$, as well as experimental correlations for dimensionless $Nu$, $Sh$ and $Eu$ numbers demonstrate an excellent symmetry. In this table, we can see that the migration mechanism of heat, mass or power was described by a constitutive relation between a physical flux and a driving force, and the transport mechanism of convection, convective heat or mass transfer was described by a dot product between velocity vector and temperature, concentration or pressure gradient. In addition, the units of migration coefficients $a$, $f$ and $D$ are all the same, i.e., m²/s. The dimensionless experimental correlations of convection, convective heat and mass transfer are all similar.

### 3. Hydrodynamic experiment for the convection

#### 3.1. Experimental setup

Dimensionless experimental correlations for the heat and mass transfer had been widely studied in the world, but seldom found for the convection in our literature survey. In order to obtain a correlation described by Eq. (24), an experimental system was established to verify hydrodynamic performance of plain tube, as shown in Fig. 4. In this system, the tube length and diameter are 3200 and...
19 mm respectively. The tube is long enough to ensure measuring accuracy for pressure differences between the inlet and outlet of the tube. A heater installed in a water tank is controlled by a voltage regulator to adjust water temperature. A frequency converter is used to control the pump and thereby adjusting water flow rate in the system.

To raise the testing accuracy, two differential pressure gauges, one (model: FCX-AIII FKCT11V5-LUCYY-BAY) with an accuracy of ±0.1% for a measuring range of 0–1 kPa and another (model: YOKOGAWA EJA 110A) with an accuracy of ±0.076% for a measuring range of 0–10 kPa, were applied respectively to measure pressure drop in the tube at different flow rate. In addition, to measure water temperature, two K-type armored thermocouples (model: NK-1031) with an accuracy of ±0.3 °C were installed at the inlet and outlet of the tube, respectively. The volumetric flow rate (model: ADMAG AXF010G) was measured by using a magnetic flow meter in a measuring range of 0–1.5 m³/h with an accuracy of ±0.2%, which was calibrated by the weighing method. All experimental data were automatically collected by a data acquisition system.

The temperature-dependent physical properties of water are presented in Table 2. The average temperature of water is calculated by

![Fig. 5. Validation of friction coefficient f in plain tube at ambient water temperature 23 °C.](image)

\[
T_f = \frac{T_{f,\text{in}} + T_{f,\text{out}}}{2}
\]

where \(T_{f,\text{in}}\) and \(T_{f,\text{out}}\) are temperatures measured by the thermocouples installed at the inlet and outlet of the tube, respectively. The experiment was conducted at different average temperatures of water in the range of 23–80 °C.

The uncertainty analysis for the experimental data was based on ANSI/ASME standard [36]. The overall uncertainty \(w_R\) can be calculated by a function of \(n\) independent variables,

\[
w_R = \left(\sum_{i=1}^{n} w_x R \left(\frac{\partial R}{\partial x_i}\right)^2\right)^{1/2},
\]

where \(R\) is a function of measured variables \(x_1, x_2, \ldots, x_n\) and \(w_{x_1}, w_{x_2}, \ldots, w_{x_n}\) are the uncertainties of measured variables. The measured variables in this experiment are pressure drop (Δp), inlet
temperature ($T_{in}$), outlet temperature ($T_{out}$) and volumetric flow rate ($Q$), and their uncertainties are calculated as:

$$w_{dp} = 0.1\% \times 1000 \text{ Pa} = 1 \text{ Pa}, \quad \Delta p < 1000 \text{ Pa},$$

$$w_{dp} = 0.076\% \times 10000 \text{ Pa} = 7.6 \text{ Pa}, \quad \Delta p > 1000 \text{ Pa},$$

$$w_{t_{in}} (w_{t_{out}}) = 0.3 \degree C,$$

$$w_{Q} = 0.2\% \times 1.5 \text{ m}^3/\text{h} = 0.003 \text{ m}^3/\text{h},$$

$$w_{T_{in}} = \left( \frac{1}{2} w_{t_{in}} \right)^2 + \left( \frac{1}{2} w_{t_{out}} \right)^2 \right)^{1/2} = 0.21 \degree C.$$

Then, according to Table 2, the uncertainties of viscosity and density of the fluid can be determined by the following formulas,

$$w_{\mu} = \left[ -9.4436 \times 10^{-4} + 2 \times 2.7020 \times 10^{-6} (T_f + 273.15) \right],$$

$$w_{p} = \left[ 1.8793 - 2 \times 3.5676 \times 10^{-3} (T_f + 273.15) \right] \right)^{1/2} \cdot \left( w_{T_f} \right) \right)^{1/2}.$$

From the definitions of $Eu$ and $Re$ numbers,

$$Eu = \frac{\Delta p}{\mu u_m^2} = \Delta p \left( \frac{u_m}{\rho} \right)^2 = \frac{\pi^2 D^4 \Delta p}{16 \rho Q^2},$$

$$Re = \frac{\rho u_m D}{\mu} = 4\rho \frac{Q}{\pi \mu D},$$

the uncertainties of $Eu$ and $Re$ numbers can be determined by

$$w_{Eu} = \left[ \left( \frac{\pi^2 D^4}{16 \rho Q^2} \right) (w_{\mu})^2 + \left( \frac{\pi^2 D^4 \Delta p}{16 \rho Q^2} \right) (w_{\rho})^2 + \left( \frac{\pi^2 D^4 \Delta p}{8 \rho Q^2} \right) (w_{Q})^2 \right]^{1/2}.$$

$$w_{Re} = \left[ \frac{4Q}{\pi \mu D} (w_{\rho})^2 + \left( \frac{4 \rho}{\pi \mu D} \right) (w_{Q})^2 + \left( -\frac{4 \rho Q}{\pi \mu D} \right) (w_{\mu})^2 \right]^{1/2}.$$

From Eq. (12), the uncertainty of $Li$ number can be determined by

$$w_{Li} = \left[ \left( -\frac{\mu}{\rho^2} \right) (w_{\mu})^2 + \left( \frac{1}{\rho^2} \right) (w_{\rho})^2 \right]^{1/2}.$$

Thus, the maximum relative uncertainties for the $Eu$, $Re$ and $Li$ numbers can be calculated as $\pm 5.68\%$, $\pm 2.56\%$ and $\pm 0.53\%$, respectively.

### 3.2. Dimensionless experimental correlation

The experimental correlations of friction coefficient $f$ for the turbulent flow in the plain tube were given by Blasius [37],

$$f = 0.3164 Re^{0.25}, \quad 4000 < Re < 10^5,$$

and Petukhov [38],

$$f = (0.79 \ln Re - 1.64)^{-2}, \quad 2300 < Re < 10^6.$$

According to the Darcy expression, we have

$$h_f = \frac{L}{d} \frac{u_m^2}{2g},$$

where $h_f = \Delta p/(\rho g)$ is hydrodynamic loss. Then we can have a relation between the pressure drop and friction coefficient,

$$\Delta p = \frac{\rho u_m^2 L}{2d} f,$$

or

$$Eu = \frac{L}{2d} f.$$
tures at different heater power were measured at different flow rate, which were used to determine kinematic viscosity and thermal conductivity of water in the $Li$ number. Thus, the $Eu$ number calculated by measured pressure drop could be fitted according to the experimental data of $Re$ and $Li$ number.

As shown in Fig. 5, the experiment at ambient water temperature 23°C was conducted to validate friction coefficient $f$ in a plain tube. The results show that the present experimental data agreed with Eqs. (34) and (35) in a maximum deviation less than 7.83%.

As shown in Fig. 6, the experiments at different water temperatures in the range of 23–80°C and Reynolds numbers in the range of 4000–57,000 were carried out to validate the experimental correlation expressed by Eq. (24). In this figure, we can observe that all experimental data of $f$ were consistent with Eqs. (34) and (35) within a deviation range of $-4.74$ to $7.83\%$, which implies that the data processing method applied in the present work has a good reliability.

Table 3

<table>
<thead>
<tr>
<th>Model</th>
<th>Grid number</th>
<th>$Nu$</th>
<th>$Eu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>339,797</td>
<td>32.8134</td>
<td>6.4316</td>
</tr>
<tr>
<td>2</td>
<td>669,851</td>
<td>29.4165</td>
<td>6.0754</td>
</tr>
<tr>
<td>3</td>
<td>1,013,215</td>
<td>29.0103</td>
<td>6.0105</td>
</tr>
</tbody>
</table>

Based on our experimental data, a dimensionless experimental correlation for the $Eu$ number versus the $Re$ and $Li$ numbers was fitted for the plain tube,

$$Eu = 32.0326Re^{0.2652}Li^{0.0025}, 4000 < Re < 57000; 20°C < T < 80°C.$$  \[39\]

As shown in Fig. 7, the values of $Eu$ number calculated by Eq. (39) agreed well with the experimental data of $Eu_{Exp}$ number with a deviation of $\pm 5.7\%$. This may suggest that the experimental method proposed in the present study could also be reasonably applied to all heat transfer enhanced tubes.

To validate the above dimensionless experimental correlation, a numerical experiment was conducted through CFD method. A circular tube with diameter of 19 mm and length of 3200 mm was applied in the numerical calculation. The ranges of Reynolds number and water temperature used in numerical experiment were 5000–55,000 and 20–80°C, respectively. Based on the numerical data of $Eu$ number versus $Re$ and $Li$ numbers, a dimensionless numerical correlation for turbulent flow in a circular tube was fitted by the MATLAB software,

$$Eu = 32.3352Re^{0.2622}Li^{0.0026}, 5000 < Re < 55000; 20°C < T < 80°C.$$  \[40\]

As shown in Fig. 8, good agreements are achieved with a deviation range from $-7.0\%$ to $1.0\%$ between the $Eu$ number calculated by Eq. (39) and the $Eu_{Num}$ number from numerical data, which indicates that the experimental correlation (39) is suitable to evaluate the hydrodynamic performance of plain tube.

### 4. Synergy analysis for convective heat transfer

#### 4.1. Physical model of an enhanced tube

A physical model of heat transfer enhanced tube inserted by a longitudinal swirl flow generator with center-connected rectangle rods was illustrated in Fig. 9. The side lengths of connective rod and inclined rods were both 2 mm. The angle of inclined rod was 45°. The pitches between inclined rods were selected as $p = 20$, 40, 60 mm. The tube diameter was 20 mm. The mathematical model including continuity, momentum and energy equations was discussed in Section 2, and a periodic boundary condition was used. The periodic length of computational domain was one pitch.

![Fig. 11. Comparisons between numerical results and theoretical values of plain tube.](image)

![Fig. 12. Numerical results of circular tube with a generator in a cross section at $p = 40$ mm, (a) tangential velocity vectors and velocity contours; (b) temperature distribution.](image)
4.2. Efficiency evaluation criterion

To evaluate the comprehensive performance of heat transfer enhanced tube, an efficiency evaluation criterion (EEC for short) was defined as

\[
EEC = \frac{Q}{Q_0} \frac{P_w}{P_{w0}}.
\]  

(41)

where \( Q \) and \( Q_0 \) are heat fluxes, and \( P_w \) and \( P_{w0} \) are power consumptions of the fluid for the heat transfer enhanced and plain tubes, respectively. If the \( Re \) number is the same in both tubes, we then have

\[
EEC = \frac{Q}{Q_0} \frac{\Delta P}{\Delta P_0}.
\]  

(42)

For different boundary conditions, Eq. (42) can be written as

\[
EEC = \frac{Nu}{Nu_0} \frac{Eu}{Eu_0}.
\]  

(43)

and simplified as

\[
EEC = \frac{Nu}{Nu_0} \frac{f}{f_0}.
\]  

(44)

Obviously, the value of \( EEC \) in Eq. (43) is more accurate than that in Eq. (44), as the \( Eu \) number reflects dimensionless pressure drop of the fluid in a tube caused by viscous dispassion and kinetic energy variation. Thus, Eq. (43) is suitable for evaluating comprehensive performance of heat transfer enhanced tube.

4.3. Numerical simulations

The grid system for computational domain generated by commercial software Gambit 2.4.6 was shown in Fig. 10. Highly refined grids near the tube wall, locally refined grids near the insert surfaces and hexahedron grids in the core region were adopted for the grid system. Mesh independence was checked by using three grid systems with different grid numbers (339,797, 669,851, and 1,013,215) at \( p = 40 \text{ mm} \) and \( Re = 900 \). The results listed in Table 3 indicated that compared with 1,013,215 elements, a grid system with 669,851 elements was sufficient for the simulations with the deviations of 1.38% and 1.07% for the \( Nu \) and \( Eu \) numbers respectively. Theoretical values of Nusselt number (4.36) and friction factor \((64/Re)\) were applied respectively to validate the performances of heat transfer and flow in the plain tube. As shown in Fig. 11, good agreements were obtained as the deviations between
the simulative and theoretical values were 4.7% for the Nusselt number and 1.1% for the friction factor.

The results of numerical simulation in a circular tube inserted with a longitudinal swirl flow generator were shown in Fig. 12. As shown in Fig. 12(a), four pairs of longitudinal swirls, which led to sufficient mixing between hot fluid near the tube wall and cold fluid in the core region, were generated in the tube. From Fig. 12(b), it can be observed that the cold fluid in the core region was guided to the tube wall, and the hot fluid near the tube wall was guided to the core region, thereby enhancing fluid disturbance and promoting temperature uniformity of the fluid. Thus, the synergy between the velocity and temperature fields was improved.

As shown in Fig. 13(a), synergy angle $\beta$ of enhanced tube was apparently lower than that of plain tube, which decreases with the decrease of the pitch. In the same time, however, synergy angle $\theta$ of enhanced tube was significantly higher than that of plain tube, which increases with the decrease of the pitch, as shown in Fig. 13(b). Compared to the plain tube, therefore, both heat transfer and power consumption were enlarged in the enhanced tube. Fig. 13(c) shows the variation of synergy angle $\gamma$ with the Re number. The magnitude of this synergy angle demonstrates how many power consumed by disturbing the fluid in an enhanced tube. From the figure we can observe that synergy angle $\gamma$ of plain tube is almost 90°, as there is no insert in the tube. However, synergy angle $\gamma$ of enhanced tube inserted by a longitudinal swirl flow generator with inclined rod pitch of 20 mm is smaller than others, due to strong fluid disturbing in the tube.

As shown in Fig. 14(a) and (b), the Nusselt numbers for the enhanced tube reached to 13.93–48.81, corresponding to 3.20–11.19 times of those for the plain tube, while the Euler numbers for the enhanced tube was in the range of 3.95–15.99, corresponding to 3.27–13.93 times of those for the plain tube. The $EEC$ values in the range of 0.74–1.12 increased with the increase of the pitch, and increased first and then decreased with the increase of the Reynolds number when the pitches were equal to 40 and 60 mm, as shown in Fig. 14(c).

4.4. Validation by PIV experiment

To validate the accuracy of the simulation model and confirm the multiple longitudinal swirls generated by center-connected rectangle rods, a measurement of flow field at cross section 10 mm downstream the generator had been carried out by stereoscopic-PIV (Particle Image Velocimetry) in this study. Fig. 15(a) and (b) show the schematic diagram of stereoscopic-PIV system and the pictures of the generator with center-
connected rectangle rods, respectively. The pitch of inclined rectangle rods is 40 mm, and the measured Reynolds number is 900 in the experiment. Except exposure time-delay that was set to 0.9 ms, other experimental parameters were chosen as the same as that in our previous PIV experiment, which could be found in Ref. [39] for more details.

Fig. 16(a)–(c) show the comparisons between numerical and experimental results. Experimental results of PIV measurement confirmed that four pairs of longitudinal swirls were generated by center-connected rectangle rods in the tube flow, which agree with the numerical results, as shown in Fig. 16(a). In addition, a quantitative comparison of intensity of longitudinal swirls between numerical and experimental results is displayed in Fig. 16(b). It is observed that the vorticities in the z-direction in numerical simulation agree well with that of experimental data. Fig. 16(c) indicates that the numerical and experimental results have a consistent distribution of the velocity in the z-direction. Therefore, the results of PIV measurement well confirmed that the numerical simulation applied in this study has a reasonable reliability, and the investigated model has a credible accuracy in predicting heat transfer performance for the enhanced tube.

5. Conclusions

In summary, we can make the following conclusions for the present study.

(1) A conservative equation of fluid mechanical energy was proved to be one of the basic governing equations for convective heat and mass transfer, which can demonstrate the physical mechanism of transport process by a constitutive relation between power flux and pressure gradient, and a dot product between velocity vector and pressure gradient. It can be further developed as a synergy equation to determine how to reduce power consumption of the fluid in a tube.

(2) Three synergy equations were introduced to reflect the synergy relations among velocity, pressure, temperature and component concentration, and three synergy angles were discussed to judge how to transfer more heat and mass, and consume less power in the transport process. For the laminar convective heat transfer in a tube inserted with a longitudinal swirl flow generator, the synergy angles $\beta$ and $\gamma$ of enhanced tube are less than that of plain tube (nearly $90^\circ$), and the synergy angle $\theta$ of enhanced tube is bigger than that of plain tube (around $10^\circ$). The $EEC$ value, that represents comprehensive performance of enhanced tube, reaches to $0.98–1.12$ for the generator with inclined rod pitch of $60 \text{ mm}$.

(3) A dimensional correlation was theoretically predicted through a synergy equation based on fluid mechanical energy conservation, and experimentally obtained as $Eu = 32.0326Re^{0.2852}Li^{0.0025}$ in the Reynolds number of
The deviation between experimental and numerical correlations is less than 7%. This may provide an available method to evaluate hydrodynamic performance for heat transfer tubes.

**Conflict of interest**

The authors do not have any possible conflicts of interest.

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