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Research Paper

Finite time thermodynamic analysis of a solar duplex Stirling refrigerator

Dongdong Dai, Zhichun Liu, Fang Yuan, Rui Long, Wei Liu*

School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

HIGHLIGHTS

- A solar duplex Stirling refrigerator system is proposed.
- A thermodynamic model considering finite time is developed.
- Polytropic process in expansion/compression is adopted.
- Two constraints including power and time are considered.

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ABSTRACT

Stirling engine is a machine that has the same theoretical efficiency as that of a Carnot cycle under ideal conditions and has an aptitude for utilizing sustainable energy such as solar and biomass energy. In this study, a solar duplex Stirling refrigerator system was proposed to capture solar energy for refrigeration. The system is composed of a solar collector for collecting solar thermal energy, a Stirling heat engine for converting thermal energy into power, and a Stirling refrigerator to utilize power for cooling. The system is modeled based on finite time thermodynamics. Moreover, two constraints i.e., power and cycle time equality of the engine and refrigerator are considered in this model. In addition, the linearized heat loss of the solar collector, finite rates of heat transfer, regenerative heat loss, and conductive thermal bridging loss of the Stirling engine and refrigerator are considered in this model. Finally, the effects of design parameters on the performance of the system are investigated. This may serve as clean technology for solar refrigeration.

1. Introduction

Renewable energy such as solar, wind, tidal, and geothermal energy contributes to a supplement to energy and a reduction in environmental pollution. Among these renewable energy, solar energy, as one of the most widely distributed energy all over the world, has been paid more attention owing to the increase in demand for energy and environmental protection. The energy consumption of the refrigeration industry has increased during the last few decades. Researches on the refrigerators driven by solar energy have aroused wide attention especially in the countries and regions with the availability of a huge amount of solar energy and long daily sunny hours.

To date, a variety of solar refrigerators have been developed. These solar refrigerators can be classified into solar thermal, solar electric, and other newly emerging technologies [1]. A solar thermal refrigeration system uses solar heat to provide energy to drive the refrigeration part. As a kind of solar thermal refrigeration technologies, thermo-mechanical refrigeration systems use Rankine or Stirling engine to convert solar heat to mechanical work for driving a vapor compression refrigerator. Sorption refrigeration including absorption [2], adsorption [3], and desiccant cooling [4] is the other kind of solar thermal refrigeration technologies. A solar electric refrigeration system is composed mainly of photovoltaic panels and an electrical refrigeration device. Technologies like vapor compression, thermoelectric elements, Stirling refrigerator, thermo-acoustic, and magnetic cooling have been used in combination with photovoltaic panels for solar electric refrigeration [1].

Besides solar electric and solar thermal refrigeration, some newly emerging technologies have been developed. A solar-powered electrochemical refrigeration system was proposed to collect the solar electrical energy to drive the electrochemical refrigerator [5]. An ejector refrigerator is powered by solar thermal energy and based upon the ejector refrigeration [6]. Besides, some combined or hybrid systems have been developed in solar refrigeration researches [7].

Stirling engines are appealing engines with regard to renewable energy utilization as they have high theoretical efficiency, low

* Corresponding author.
E-mail address: w_liu@hust.edu.cn (W. Liu).

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Nomenclature

A	collector aperture area, m^2
C	collector concentrating ratio
c_v	isochoric heat capacity of the working fluid, J/(g·K)
c_1, c_1, c_1, c_1	constant
e_r	regenerator effectiveness of Stirling engine
$e_{r'}$	regenerator effectiveness of Stirling refrigerator
h	conduction/convection coefficient, W/(m^2·K)
I	direct solar flux intensity, kW/m^2
k	thermal leak coefficient, W/K
M	constant rate of regeneration in engine, s/K
M'	constant rate of regeneration in refrigeration, s/K
n	mole number of working fluid, mole
P	output power of engine, W
P'	power consumption of refrigerator, W
q	cooling rate, W
Q	heat, J
R	ideal gas constant, J/(mol·K)
t	time, s
T	temperature, K
V	volume, m^3
W	work, J

Greek symbols

δ	Stefan's constant, W/(m^2·K^4)
ε	emissivity factor of the collector
η_0	collector optical efficiency
η_e	thermal efficiency of Stirling engine
η_s	thermal efficiency of dish collector
λ	volume ratio of engine
λ'	volume ratio of refrigerator
σ_R	coefficient of performance
σ_o	overall coefficient of performance
τ	cyclic period of engine, s
τ'	cyclic period of refrigerator, s

Subscripts

0	ambient
app	collector aperture
e	engine
h	expansion process in engine
h'	expansion process in refrigerator
H	heat source
H'	heat sink of refrigerator
l	compression process in engine
l'	compression process in refrigerator
L	heat sink
L'	heat source of refrigerator
r	regeneration
rec	absorber
u	useful gain of dish solar collector

emissions, low noise, and multi-fuel capability. Stirling engines driven by solar energy facilitates the conversion of solar energy into power, which is environmentally friendly owing to its nature. Stirling engine-based units are considered best among the most effective low-power range solar thermal conversion units [8]. As mentioned above, Stirling refrigerators have also been utilized by numerous engineering companies in refrigeration devices. A duplex Stirling refrigerator is an integrated refrigerator that is composed of a Stirling heat engine and a Stirling refrigerator used for cooling. It is a promising device owing to its high efficiency, reliability, and compactness, and low noise, emission, and operation cost [9]. Certain researchers studied the duplex Stirling refrigerator for domestic refrigeration applications [10,11]. As the Stirling engine is competitive in solar energy systems, the duplex Stirling refrigerator can also be considered in solar refrigeration.

In analyzing and designing Stirling engines, researchers proposed several models using classical thermodynamics [12–14]. With the development of finite-time thermodynamics (FTT) [15,16], several Stirling engine models have been proposed using FTT. Considering the irreversibility in the external heat transfer processes, Blank et al. [17] analyzed an endoreversible Stirling engine with FFT and obtained the efficiency at maximum power. Wu et al. [18,19] investigated the optimal performance of a Stirling engine with finite-speed effect in the regenerating processes and finite heat transfer in the isothermal processes. Analyzing the irreversibility of regenerator caused by temperature difference, Dai et al. [20] obtained the theoretical limits of regenerator and Stirling engines via FTT. Using the FTT method, Kaushik et al. [21,22] developed a Stirling heat engine model subjects to a finite heat capacitance rate of working substance, the heat leak between two reservoirs, and regenerative losses. Dai et al. [23] constructed a refined model considering the regenerative loss that was supplied by the heat source and optimized the Stirling engine with a multi-objective optimization method based on MOPSOCD. Ahmadi et al. [24–26] optimized and compared the finite time thermodynamic model of Stirling engine with other thermal models.

Stirling engines driven by solar energy have also been investigated. Kongtragool et al. [27] presented a theoretical investigation regarding the optimum absorber temperature of a once-reflecting full conical solar concentrator for maximizing the total efficiency of a solar Stirling engine. Considering the limited heat transfer and regenerative loss, Chen et al. [28] investigated the performance of a combined system consisted of a Stirling engine and a solar collector using FTT. Li et al. [29] applied FTT to optimize the maximum power output and the corresponding thermal efficiency of a solar Stirling engine. Ahmadi et al. [30–33] adopted a multi-objective optimization method in designing the solar Stirling engines and obtained considerably realistic results compared with the single-objective optimization method.

The theory of FTT has also been successfully used in the analysis of Stirling refrigerators. With a given cooling rate, Chen et al. [34] developed an irreversible Stirling refrigerator model that considered the finite rate heat transfer, regenerative loss, and heat leak. Tyagi et al. [35] proposed an irreversible Stirling cryogenic refrigerator model including external and internal irreversibilities along with finite heat capacities of external reservoirs. They also optimized the Stirling refrigerator model that they proposed with the thermoeconomic criterion [36]. Based on NSGA-II algorithm, Ahmadi et al. [37] optimized the input power, coefficient of performance, and cooling rate of a Stirling refrigerator simultaneously using the method of multi-objective evolutionary approaches. Combining the second version of non-dominated sorting genetic algorithm (NSGA-II), Varun [38] developed a multi-objective optimization for three Stirling models.

Only a few thermodynamic models of the duplex Stirling refrigerator are found in the literature. Erbay et al. [39] conducted a thermodynamic analysis of a duplex Stirling refrigerator considering its engine and refrigerator separately but under the constraint of the work equality. According to the advantages of solar Stirling engines and duplex Stirling refrigerators, we proposed a solar duplex Stirling refrigerator system consisting of a solar collector and a duplex Stirling refrigerator for converting solar heat energy for cooling indirectly. Unlike the previous thermodynamic analysis under the only constraint of the work equality, we took finite time into consideration and two constraints, power and cycle time equality, were considered as the constraints of the duplex system. The proposed model was analyzed using the FTT method, resulting in a considerably realistic investigation of the performance of the system. In addition, we studied the effects of the design parameters on the overall performance of the solar duplex Stirling refrigerator system for application guidance.

2. Thermodynamic analysis of solar duplex Stirling refrigerator system

The solar duplex Stirling refrigerator is a system for converting solar energy for indirect cooling. As an integrated system, the solar duplex Stirling refrigerator consists of a dish solar collector for collecting solar heat energy, a Stirling cycle engine for converting solar heat energy to work, and a Stirling cycle refrigerator for cooling. The *T-S* diagram of the solar duplex Stirling refrigerator system is depicted in Fig. 1. A Stirling cycle consists of two isothermal branches and two isochoric regenerating branches. As finite-time thermodynamics indicates, within a finite time, there are temperature differences between the working fluid and external heat reservoirs. The working fluid of the Stirling engine works between the heat source with constant temperature T_H and heat sink with constant temperature T_L. The working fluid of the Stirling refrigerator absorbs heat from the heat source with constant temperature $T_{L'}$, and rejects heat to the heat sink with constant temperature $T_{H'}$. The power and cycle time are two constraints of the solar duplex refrigerator system; the produced power by the engine is consumed by the refrigerator, and the cycle time of the Stirling engine is equivalent to that of the Stirling refrigerator. The mathematical model of the duplex Stirling refrigerator system is performed considering the three components of the duplex system separately as the dish solar collector, the heat engine, and the refrigerator sides on finite-time thermodynamics.

2.1. Analysis of dish solar collector

The dish solar collector employed in this study is a device that can track the sun, focus solar energy into a cavity absorber, and convert solar heat energy to thermal energy for heating expansion space of the Stirling engine. According to the previous research [29,33], actual useful heat gain of the dish solar collector considering conduction, convection, and radiation losses is given by

$$Q_u = IA_{app}\eta_0 - A_{rec}[h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4)] \tag{1}$$

where I is the direct solar flux intensity, A_{app} is the collector aperture area, η_0 is the collector optical efficiency, A_{rec} is the absorber area, h is conduction/convection coefficient, T_H is the absorber temperature, T_0 is the ambient temperature, ε is emissivity factor of the collector, δ is the Stefan's constant.

Thermal efficiency η_s of the dish collector is defined as the ratio of the useful collected energy to the solar energy input

$$\eta_s = \frac{Q_u}{IA_{app}} = \eta_0 - \frac{1}{IC}[h(T_H - T_0) + \varepsilon\delta(T_H^4 - T_0^4)] \tag{2}$$

where $C = A_{app}/A_{rec}$ is the collector concentrating ratio. With a given η_s, the absorber temperature T_H can be obtained from Eq. (2) with different direct solar flux intensities.

2.2. Analysis of Stirling heat engine

In practice, regenerative heat loss should be considered owing to the imperfect regenerator. The regenerative effectiveness, defined as the ratio of heat transfer in real to that in ideal during regenerative processes, can be determined using the following expression

$$e_r = \frac{Q_{real}}{Q_{ideal}} = \frac{T_3 - T_2}{T_4 - T_2} = \frac{T_4 - T_1}{T_4 - T_2} \tag{3}$$

Denote the ratio of working fluid temperature in the Stirling engine as

$$\gamma = \frac{T_2}{T_4} \tag{4}$$

Subsequently, the working fluid temperature in state 1, 2, and 3 can be expressed as

$$\begin{cases} T_1 = [1 - (1 - \gamma)e_r]T_4 \\ T_2 = \gamma T_4 \\ T_3 = [(1 - \gamma)e_r + \gamma]T_4 \end{cases} \tag{5}$$

It is necessary for the Stirling engine to proceed within finite time, and thus, the heat transfer between the external heat reservoirs and working substance occurs with a finite temperature difference. From Section 2.1 we can obtain the absorber with temperature T_H, which is used as the heat source of Stirling heat engine. The working fluid of Stirling heat engine expands with variable temperature owing to the finite time and convection heat transfer between the working fluid and constant temperature heat source. Instead of an isothermal process, a polytropic process is adopted here to describe the expansive process. Denote the polytropic exponent by n_H and the state equation of the working fluid in the expansive process can be presented as

$$pV^{n_H} = p_3V_3^{n_H} = p_4V_4^{n_H} = c_1 \tag{6}$$

where c_1 is a constant. Similarly, the state equation of the working fluid in the compressive process can be stated as

$$pV^{n_C} = p_1V_1^{n_C} = p_2V_2^{n_C} = c_2 \tag{7}$$

where n_C and c_2 are the polytropic exponent and constant in the compressive process, respectively.

Combining the equations above and the ideal gas state equation, we can obtain the expression of polytropic exponents as

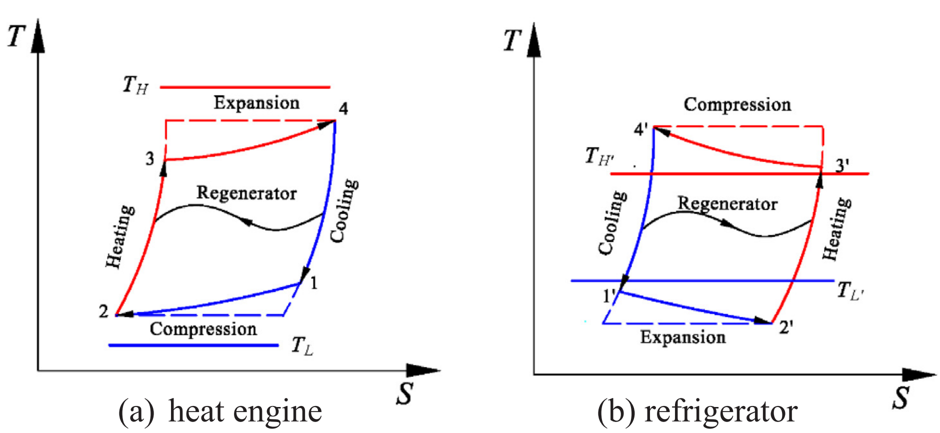

Fig. 1. *T-S* diagram of the solar duplex Stirling refrigerator system.

$$n_H = \frac{\ln(e_r + (1 - e_r)\gamma)}{\ln\lambda} + 1 \quad (8)$$

and

$$n_C = \frac{\ln\left(\frac{\gamma}{(1-e_r)+e_r\gamma}\right)}{\ln\lambda} + 1 \quad (9)$$

where λ is the ratio of volume during the regenerative processes, i.e.,

$$\lambda = \frac{V_1}{V_2} = \frac{V_4}{V_3} \quad (10)$$

Denoting the variable temperature of working fluid by T, the heat absorbed from the absorber and released to the heat sink can be described as

$$\delta Q_h = \alpha_h (T_H - T)dt \quad (11)$$

and

$$\delta Q_l = \alpha_l (T - T_L)dt \quad (12)$$

where α_h is high-temperature side convection heat transfer coefficient, α_l is the low-temperature side convection heat transfer coefficient, the heat sink temperature is T_L.

The energy conservation equations in expansive and compressive processes can be expressed as

$$\delta Q_h = mc_v dT + pdV \quad (13)$$

and

$$-\delta Q_l = mc_v dT + pdV \quad (14)$$

where c_v is the isochoric heat capacity of the working fluid.

From Eqs. (11)–(14) we can obtain the formulas in expansive and compressive processes as

$$dt = \frac{c_1\left(\frac{c_v}{R_g}(1 - n_H) + 1\right)}{\alpha_h\left(T_H V^{n_H} - \frac{c_1 V}{mR_g}\right)}dV \quad (15)$$

and

$$dt = \frac{c_2\left(\frac{c_v}{R_g}(1 - n_C) + 1\right)}{\alpha_l\left(T_L V^{n_C} - \frac{c_2 V}{mR_g}\right)}dV \quad (16)$$

The expansion and compression durations can be derived by integrating Eqs. (15) and (16) as follows

$$t_h = \frac{m(R_g + c_v(1 - n_H))}{\alpha_h(n_H - 1)}\ln\frac{mR_g T_H - c_1 V_4^{1-n_H}}{mR_g T_H - c_1 V_3^{1-n_H}} \quad (17)$$

and

$$t_l = \frac{m(R_g + c_v(1 - n_C))}{\alpha_l(n_C - 1)}\ln\frac{mR_g T_L - c_2 V_2^{1-n_C}}{mR_g T_L - c_2 V_1^{1-n_C}} \quad (18)$$

where t_h indicates the expansion duration and t_l indicates the compression duration.

As a finite-speed effect exists in the regenerating processes [18], the time of the regeneration can be assumed to be proportional to the temperature difference of the working fluid in the two isothermal processes, i.e.,

$$t_{r1} = t_{r2} = M(T_4 - T_2) \quad (19)$$

where M is the constant rate of the two regenerating processes. For a non-uniform temperature change, M is the average rate of temperature change.

From Eqs. (17)–(19), the duration of each branch of the Stirling engine cycle can be obtained, and thus, the cyclic period τ can be obtained as

$$\tau = t_h + t_l + t_{r1} + t_{r2} \quad (20)$$

Work produced during expansive and compressive processes can be expressed as follows

$$W_h = \int_{V_3}^{V_4} pdV = \frac{mR_g T_4}{1 - n_H}(1 - \gamma)(1 - e_r) \quad (21)$$

and

$$W_l = \int_{V_1}^{V_2} pdV = \frac{mR_g T_4}{n_C - 1}(1 - \gamma)(1 - e_r) \quad (22)$$

As no work is produced during regenerative processes, the cycle work of the Stirling engine cycle can be determined as

$$W = W_h + W_l = \left(\frac{1}{1 - n_H} - \frac{1}{1 - n_C}\right)mR_g T_4(1 - \gamma)(1 - e_r) \quad (23)$$

In the expansive process, the heat absorbed from the external reservoir is used to produce work and increase the internal energy of the working fluid, which can be expressed as

$$Q_h = W_h + \Delta U = (1 - \gamma)(1 - e_r)\left(\frac{R_g}{1 - n_H} + c_v\right)mT_4 \quad (24)$$

For a Stirling heat engine, a conductive thermal leaking loss exists between the two external heat reservoirs. The conductive thermal leaking loss can be assumed to be proportional to the cycle time and can be expressed as

$$Q_{h\_leak} = k_{h\_leak}(T_H - T_L)\tau \quad (25)$$

where k_{h_leak} represents the thermal leak coefficient between the absorber and the heat sink.

The heat released from the absorber is absorbed by the working fluid and consumed via conductive thermal leaking loss. As a result, the net heat released from the absorber Q_H and absorbed by the heat sink Q_L is given by

$$Q_H = Q_h + Q_{h\_leak} \quad (26)$$

Therefore, the power output can be obtained by dividing the work output by the cycle period, i.e.,

$$P = \frac{W}{\tau} = \frac{\left(\frac{1}{1-n_H} - \frac{1}{1-n_C}\right)mR_g T_4(1 - \gamma)(1 - e_r)}{t_h + t_l + 2t_r} \quad (27)$$

The thermal efficiency of the Stirling heat engine can be expressed as

$$\eta_e = \frac{W}{Q_h + Q_{h\_leak}} = \frac{\left(\frac{1}{1-n_H} - \frac{1}{1-n_C}\right)mR_g T_4(1 - \gamma)(1 - e_r)}{(1 - \gamma)(1 - e_r)\left(\frac{R_g}{1-n_H} + c_v\right)mT_4 + k_{h\_leak}(T_H - T_L)\tau} \quad (28)$$

2.3. Analysis of Stirling refrigerator

Regenerative heat loss should be considered owing to the imperfect regenerator in the Stirling refrigerator. As shown in Fig. 1(b), the regenerative effectiveness of the Stirling refrigerator can be defined as

$$e_{r'} = \frac{Q'_{real}}{Q'_{ideal}} = \frac{T_{3'} - T_{2'}}{T_{4'} - T_{2'}} = \frac{T_{4'} - T_{1'}}{T_{4'} - T_{2'}} \quad (29)$$

The temperature ratio and volume ratio can be denoted as

$$\gamma' = \frac{T_{2'}}{T_{4'}} \quad (30)$$

and

$$\lambda' = \frac{V_{2'}}{V_{1'}} = \frac{V_{3'}}{V_{4'}} \quad (31)$$

Subsequently, the temperature of working fluid in states 1′, 2′, and 3′ can be represented as

$$\begin{cases} T_{1'} = [1 - e_{r'}(1-\gamma')]T_{4'} \\ T_{2'} = \gamma' T_{4'} \\ T_{3'} = [e_{r'}(1-\gamma') + \gamma']T_{4'} \end{cases} \tag{32}$$

In a Stirling refrigerator, the working fluid absorbs heat from the heat source at a low temperature $T_{L'}$, and rejects heat to the heat sink at a high temperature $T_{H'}$. Owing to the finite time and finite temperature difference between the working fluid and external reservoirs, the temperature of the working fluid is different from that of the heat sink or heat source. Instead of isothermal processes, polytropic processes are adopted here to describe the expansive and compressive processes. Denoting the polytropic exponents by $n_{H'}$ and $n_{C'}$, the state equations of the working fluid in expansive and compressive processes can be expressed as

$$pV^{n_{C'}} = p_{3'}V_{3'}^{n_{C'}} = p_{4'}V_{4'}^{n_{C'}} = c_3 \tag{33}$$

and

$$pV^{n_{H'}} = p_{1'}V_{1'}^{n_{H'}} = p_{2'}V_{2'}^{n_{H'}} = c_4 \tag{34}$$

where c_3 and c_4 are two constants.

According to Eqs. (29)–(34), the expressions of polytropic exponents in expansive and compressive processes can be represented as follows

$$n_{C'} = \frac{\ln \frac{1}{e_{r'}(1-\gamma')+\gamma'}}{\ln \lambda'} + 1 \tag{35}$$

and

$$n_{H'} = \frac{\ln \frac{1-e_{r'}(1-\gamma')}{\gamma'}}{\ln \lambda'} + 1 \tag{36}$$

Denoting the variable temperature of the working fluid by T, the heat absorbed from the absorber and released to the heat sink can be described as

$$\delta Q_{h'} = \alpha_{h'}(T - T_{H'})dt \tag{37}$$

and

$$\delta Q_{l'} = \alpha_{l'}(T_{L'} - T)dt \tag{38}$$

where $\alpha_{h'}$ is high-temperature side convection heat transfer coefficient, $\alpha_{l'}$ is the low-temperature side convection heat transfer coefficient, the heat sink temperature is $T_{H'}$ and the heat source temperature is $T_{L'}$.

The energy conservation equations in expansive and compressive processes can be expressed as

$$-\delta Q_{h'} = mc_v dT + pdV \tag{39}$$

and

$$\delta Q_{l'} = mc_v dT + pdV \tag{40}$$

From Eqs. (37)−(40) we can obtain the formulas in expansive and compressive processes as

$$dt = \frac{(1-n_{C'})\frac{c_v}{R_g} + 1}{\alpha_{h'}\left(\frac{T_{H'}V^{n_{C'}}}{c_3} - \frac{V}{mR_g}\right)}dV \tag{41}$$

and

$$dt = \frac{(1-n_{H'})\frac{c_v}{R_g} + 1}{\alpha_{l'}\left(\frac{T_{L'}V^{n_{H'}}}{c_4} - \frac{V}{mR_g}\right)}dV \tag{42}$$

Denoting the compression duration by $t_{h'}$ and expansion duration by $t_{l'}$, the following expressions can be derived as

$$t_{h'} = \frac{m((1-n_{C'})c_v + R_g)}{\alpha_{h'}(n_{C'}-1)} \ln \frac{T_{H'}mR_g - c_3V_4^{1-n_{C'}}}{T_{H'}mR_g - c_3V_3^{1-n_{C'}}} \tag{43}$$

$$t_{l'} = \frac{m((1-n_{H'})c_v + R_g)}{\alpha_{l'}(n_{H'}-1)} \ln \frac{mR_gT_{C'} - c_4V_2^{1-n_{H'}}}{mR_gT_{C'} - c_4V_1^{1-n_{H'}}} \tag{44}$$

Since a finite-speed effect exists in the regenerating processes, the time of the regeneration in the refrigerator cycle can be assumed to be proportional to the temperature difference of the working fluid in the two isothermal processes, i.e.,

$$t_{r1'} = t_{r2'} = M'(T_{4'} - T_{2'}) \tag{45}$$

where M' is the constant rate of the two regenerating processes in refrigeration. For a non-uniform temperature change, M' is the average rate of temperature change. Subsequently, we can obtain the cyclic period τ' as

$$\tau' = t_{h'} + t_{l'} + t_{r1'} + t_{r2'} \tag{46}$$

Work produced in expansive and compressive processes can be described as

$$W_{h'} = \int_{V_3}^{V_4} pdV = \frac{1-\lambda^{1-n_{C'}}}{1-n_{C'}}mR_gT_{4'} \tag{47}$$

$$W_{l'} = \int_{V_{1'}}^{V_{2'}} pdV = \frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}mR_gT_{2'} \tag{48}$$

As no work is produced during regenerative processes, the cycle work required in the Stirling refrigerator cycle can be determined as follows

$$W_R = W_{l'} - W_{h'} = \left(\frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}\gamma' - \frac{1-\lambda'^{1-n_{C'}}}{1-n_{C'}}\right)mR_gT_{4'} \tag{49}$$

Applying the first law of thermodynamics to the process 1′−2′, the heat absorbed from the external heat source is used to produce work and increase the internal energy of the working fluid, which can be expressed as

$$Q_{l'} = W_{l'} + \Delta U = \frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}mR_gT_{l'} - mc_v(1-e_r)(T_{4'} - T_{2'}) \tag{50}$$

A conductive thermal leaking loss also exists between the two external heat reservoirs in the Stirling refrigerator cycle. The conductive thermal leaking loss can be assumed to be proportional to the cycle time and can be expressed as

$$Q_{r\_leak} = k_{r\_leak}(T_{H'} - T_{L'})\tau' \tag{51}$$

where k_{r_leak} is the heat leak coefficient between the heat sink and the cooled space.

Because of the conductive thermal leaking loss, the total heat absorbed from the cooled space $Q_{L'}$ in a cycle are given by

$$Q_{L'} = Q_{l'} - Q_{r\_leak} \tag{52}$$

Therefore, the power input, cooling rate and coefficient of performance of the Stirling refrigerator are obtained as follows

$$P' = \frac{W_R}{\tau'} = \frac{\left(\frac{1-\lambda'^{1-n_{C'}}}{1-n_{C'}} - \frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}\gamma'\right)mR_gT_{4'}}{t_{h'} + t_{l'} + t_{r1'} + t_{r2'}} \tag{53}$$

$$q = \frac{\frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}mR_g\gamma'T_{4'} - mc_vT_{4'}(1-e_{r'})(1-\gamma') - k_{r\_leak}(T_{H'} - T_{L'})\tau'}{t_{h'} + t_{l'} + t_{r1'} + t_{r2'}} \tag{54}$$

$$\sigma_R = \frac{Q_{L'}}{W_R} = \frac{\frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}} mR_g\gamma' T_{4'} - mc_v T_{4'}(1-e_{r'})(1-\gamma') - k_{r\_leak}(T_{H'}-T_{L'})\tau'}{\left(\frac{1-\lambda^{1-n_{C'}}}{1-n_{C'}} - \frac{1-\lambda'^{n_{H'}-1}}{1-n_{H'}}\gamma'\right) mR_g T_{4'}} \quad (55)$$

2.4. Analysis of entire system

In the solar duplex Stirling refrigerator system, the work generated in the Stirling heat engine provides input work for the Stirling refrigerator simultaneously. As power can be calculated by dividing work by cycle time, the power and cycle time are considered as the two constraints of the solar duplex refrigerator system; the produced power by the engine is consumed by the refrigerator, and the cycle time of the Stirling engine is equivalent to that of the Stirling refrigerator. Here we consider the maximum power of the Stirling heat engine as the input power of the Stirling refrigerator, thus the following constraints are obtained

$$P = P' \quad (56)$$

$$\tau = \tau' \quad (57)$$

With certain given parameters, we can obtain the temperature of the working fluid in state 2′ and 4′ according to the equations above.

To evaluate the performance of the entire system, the overall coefficient of performance of the system is adopted and defined as

$$\sigma_O = \eta_s \eta_e \sigma_R \quad (58)$$

3. Results and discussion

The solar duplex Stirling refrigerator, consisting of a dish solar collector, a Stirling heat engine, and a Stirling refrigerator, is an environment-friendly system that absorbs solar heat energy for cooling. Certain criteria are applied in this study for evaluation: the output power is applied to evaluate the performance of the Stirling engine; the coefficient of performance is applied to evaluate the performance of the Stirling refrigerator; the cooling rate and overall coefficient of performance are applied to evaluate the performance of the entire system.

To evaluate the effects of the design parameters on the performance of the solar duplex Stirling refrigerator system, the following parameters remain constant unless specifically stated to vary [29,36]: $n = 1$ mol, $\alpha_h = \alpha_l = 200$ W K^{-1}, $\lambda = \lambda' = 2$, $R = 8.314$ J mol^{-1} K^{-1}, $c_v = 3.214$ J g^{-1} K^{-1}, $\varepsilon = 0.9$, $k_{h\_leak} = k_{r\_leak} = 2.5$ W K^{-1}, $C = 1300$, $\alpha_{h'} = \alpha_{l'} = 700$ W K^{-1}, $\eta_0 = 0.9$, $T_L = 290$ K, $T_{L'} = 200$ K, $T_{H'} = T_0 = 288$ K, $h = 20$ W m^{-2} K, $\delta = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}, $M = M' = 1 \times 10^{-5}$ s K^{-1}.

To observe the effect of the collector optical efficiency on the performance of the system, we calculate the power P, cooling rate q, coefficient of performance σ_R, and overall coefficient of performance σ_o for different optical efficiencies and the result is shown in Fig. 2. As observed from Fig. 2, with the variation of the collector optical efficiency from 88% to 94%, the power of the engine and the overall coefficient of performance of the system increase; the cooling rate increases from 1.32 kW reaching a peak at η_0 with 91%, and then decreases to 1.39 kW; the coefficient of performance of the refrigerator increases from 0.495 to 0.522 with η_0 increases from 0.88 to 0.91, and then decreases to 0.508 with η_0 increasing to 0.94. Therefore, a large collector optical efficiency is favorable for engine power and overall coefficient of performance of the system, but an appropriate value of collector optical efficiency is required to maximize the cooling rate and the coefficient of performance of the Stirling refrigerator.

The effects of low-temperature side convection heat transfer coefficient of Stirling engine α_l on the power of Stirling engine P, cooling rate q, coefficient of performance σ_R, and overall coefficient of performance σ_o are plotted in Fig. 3. As can be observed, increasing the low temperature side convection heat transfer coefficient of Stirling engine from 200 W K^{-1} to 1200 W K^{-1} increases the power of the Stirling engine from 7985 W to 11,002 W with a high speed, and then increases to 12,897 W slowly; the overall coefficient of performance increases from 15.20% to 15.63% rapidly with α_l increasing from 200 W K^{-1} to 500 W K^{-1}, and then increases to 15.81% with a low speed for α_l larger than 500 W K^{-1}; the cooling rate remains constant at 1576 W; the coefficient of performance of the refrigerator remains constant at 0.5215. Thus, the low-temperature side convection heat transfer coefficient of Stirling engine has no effect on the cooling rate and the coefficient of performance of the refrigerator, but for a satisfactory performance with regard to the power and overall coefficient of performance, a large low-temperature side convection heat transfer coefficient of Stirling engine is required.

Fig. 4 shows the effect of high-temperature side convection heat transfer coefficient of Stirling engine α_h on the power P, cooling rate q, coefficient of performance σ_R, and overall coefficient of performance σ_o. As shown in Fig. 4, α_h has a similar effect to α_l: increasing α_h from

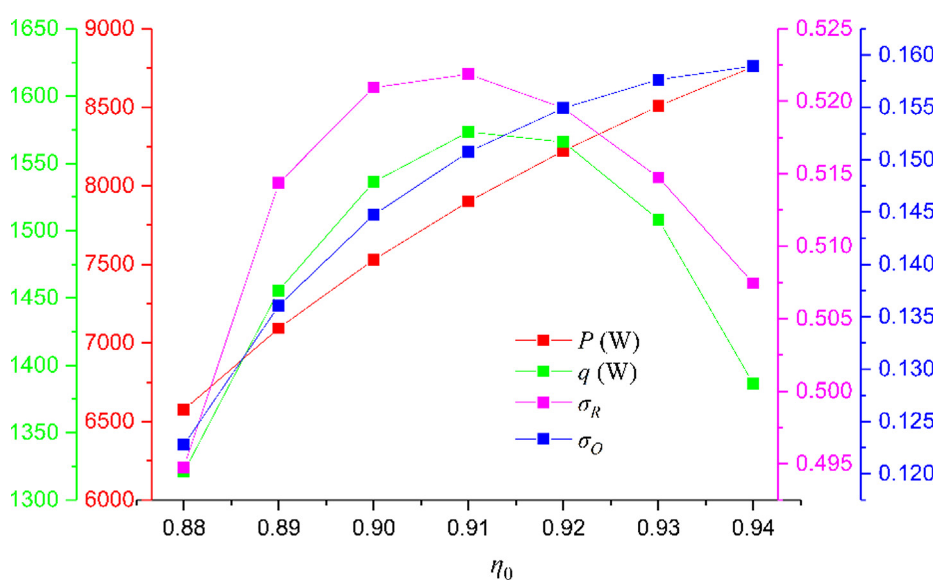

Fig. 2. Optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance versus collector optical efficiency.

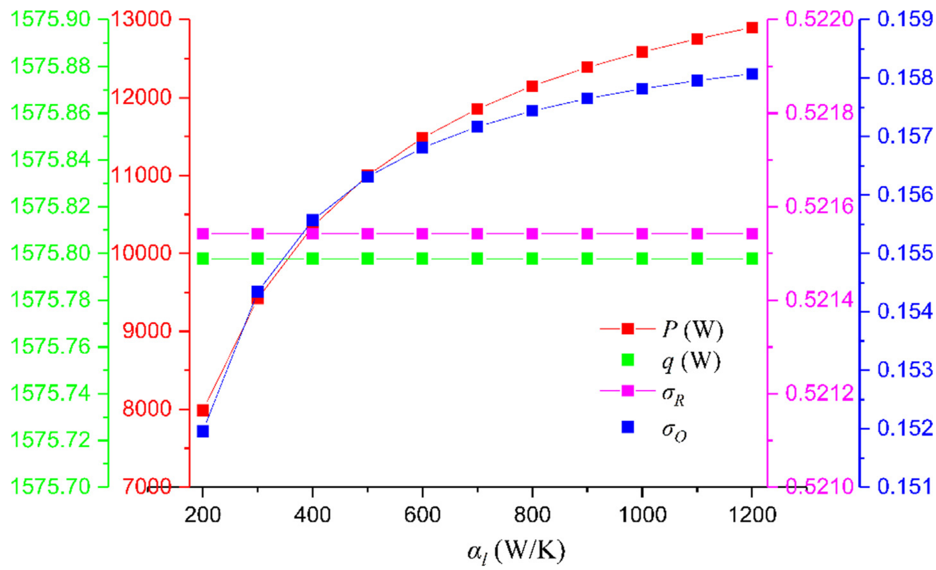

Fig. 3. Optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance versus low-temperature side convection heat transfer coefficient of Stirling engine.

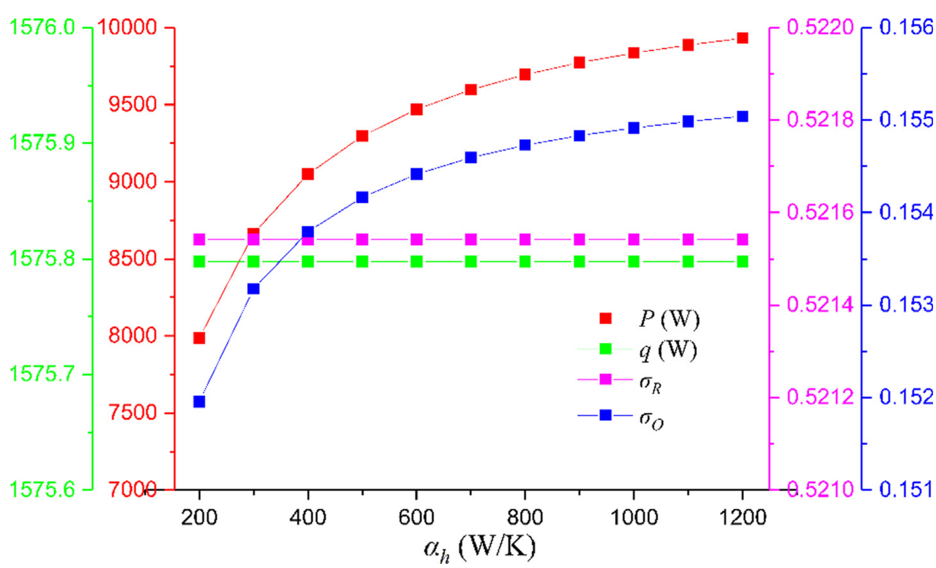

Fig. 4. Optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance versus high-temperature side convection heat transfer coefficient of Stirling engine.

200 W K^{-1}, to 1200 W K^{-1} increases the power of the Stirling engine rapidly from 7986 W to 9297 W, and then slowly to 9932 W; the overall coefficient of performance increases rapidly from 15.20% to 15.42% at first, and then to 15.50% gradually; the cooling rate and the coefficient of the refrigerator remain at 1576 W and 0.5215, respectively. Therefore, for a satisfactory performance of the present system, a large coefficient α_h is required to improve the power of the Stirling engine and the overall coefficient of performance.

The effects of thermal conductance between the heat external reservoirs and working fluid of the refrigerator on the performance of the solar duplex Stirling refrigerator system are plotted in Figs. 5 and 6. It can be observed that the thermal conductance $\alpha_{h'}$ and $\alpha_{l'}$ have no effect on the optimum power of Stirling engine. The cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance increase rapidly from 1544 W to 1923 W, 0.5174 to 0.5204, and 15.65% to 15.74%, respectively, with the thermal conductance $\alpha_{h'}$ increasing from 200 W K^{-1} to 600 W K^{-1}, and then slowly to 2121 W, 0.5212, and 15.76%, respectively, with $\alpha_{h'}$ increases to 1200 W K^{-1}. The cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance increase rapidly from 1544 W to 2480 W, 0.5174 to 0.5307, and 15.65% to 16.05%, respectively, with the thermal conductance $\alpha_{l'}$ increasing from 200 W K^{-1} to 400 W K^{-1}, and then slowly to 4329 W, 0.5411, and 16.36%, respectively, with $\alpha_{l'}$ increasing to 1200 W K^{-1}. Therefore, for a satisfactory performance of the present system, large values of thermal conduction between the heat reservoirs and working fluid of the refrigerator can improve the performance of the solar duplex Stirling refrigerator system.

4. Conclusions

In this paper, a solar duplex Stirling refrigerator system for converting solar heat energy for cooling was proposed. To develop a thermodynamic model of the system, the dish solar collector, Stirling heat engine, and Stirling refrigerator were analyzed using finite time thermodynamics. The linearized heat loss of the solar collector, finite rates of heat transfer, regenerative heat loss, and conductive thermal bridging loss of the Stirling engine and refrigerator were considered in this model. Different from previous papers that considered only one

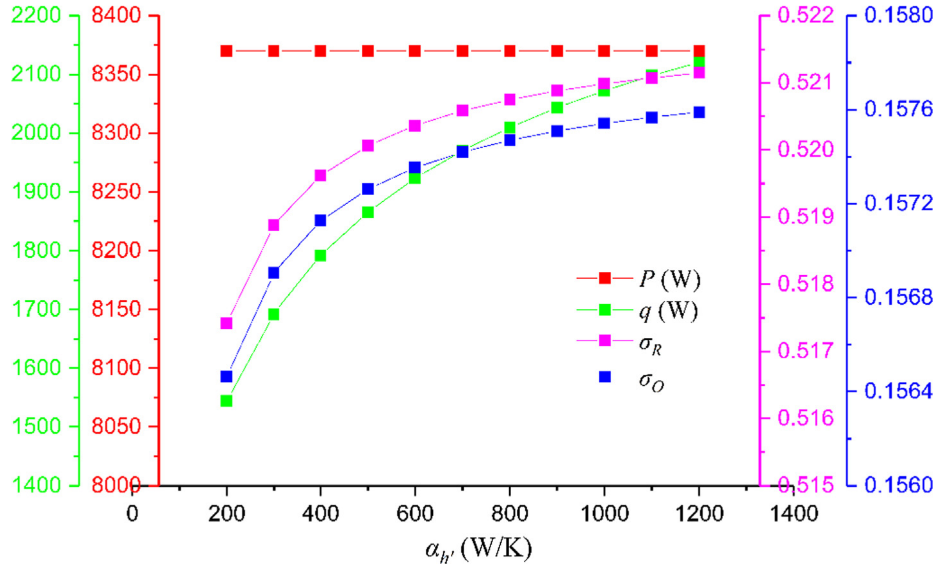

Fig. 5. Optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance versus thermal conductance between the heat sink and working fluid of refrigerator.

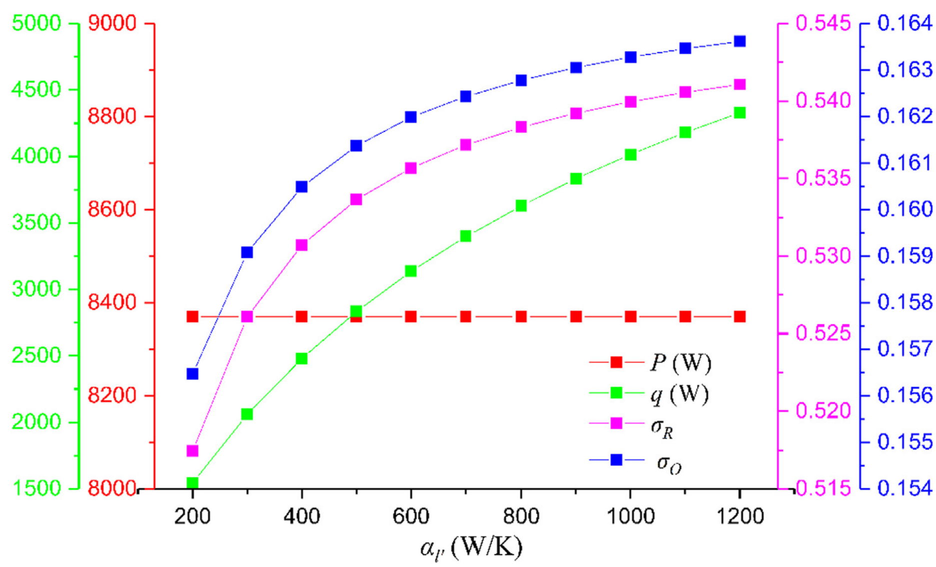

Fig. 6. Optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance versus thermal conductance between the heat source and working fluid of refrigerator.

constraint of work equality for the duplex Stirling refrigerator, we considered finite time and obtained two constraints, power and cycle time equalities, resulting in a considerably reasonable system model.

Based on the proposed model, the effects of the collector optical efficiency, high and low temperature side convection heat transfer coefficients of Stirling engine, and thermal conductance between the external heat reservoirs and working fluid of refrigerator on the performance of the solar duplex Stirling refrigerator system were investigated. Criteria such as the optimum power, cooling rate, coefficient of performance of Stirling refrigerator, and overall coefficient of performance of the system were adopted to evaluate the performance of the system. The results obtained in this study may provide certain theoretical basis for the heat management of a real solar duplex Stirling refrigerator system.

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